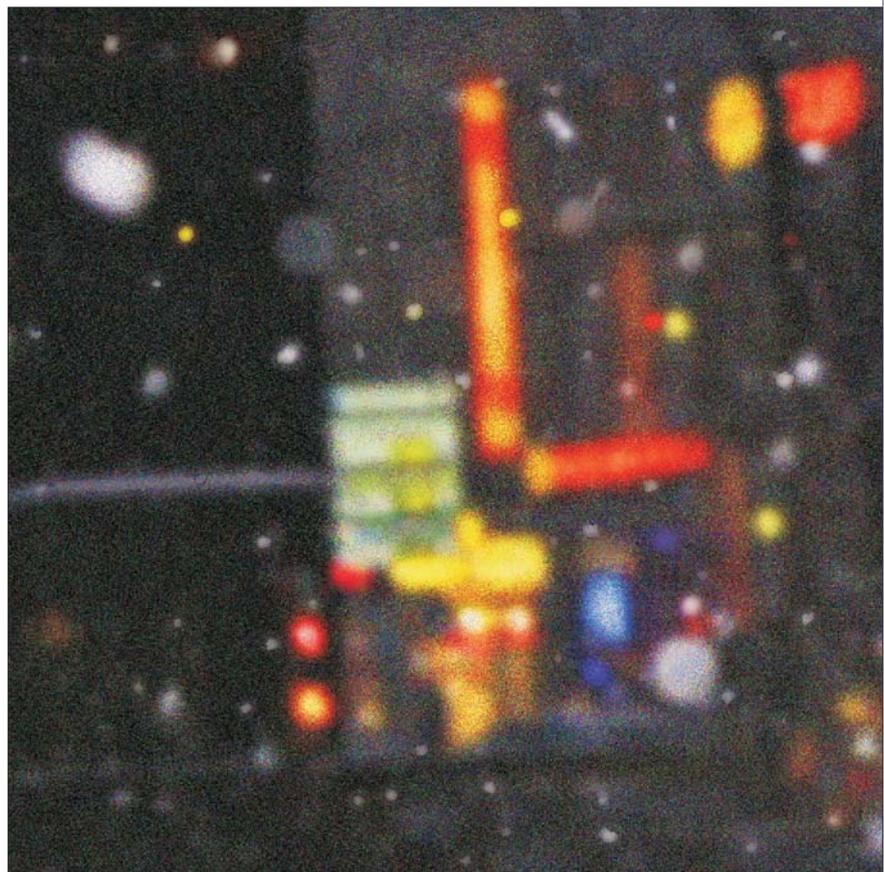


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## **ADVANTAGES OF FIXED EXCHANGE RATE REGIME FROM A GENERAL EQUILIBRIUM PERSPECTIVE**

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*ABBREVIATIONS*

DSGE – dynamic stochastic general equilibrium  
 SDR – Special Drawing Rights  
 ERM II – Exchange Rate Mechanism II  
 EMU – Economic and Monetary Union  
 CES – constant elasticity of substitution  
 GDP – gross domestic product  
 CPI – consumer price index  
 PPP – purchasing power parity  
 AR – autoregressive  
 IS – investment and saving equilibrium  
 EU – European Union

**ABSTRACT**

In this paper we estimate a small open economy DSGE model for Latvia following Lubik and Schorfheide (2007) using Bayesian methods. The estimates of the structural parameters fall within plausible ranges. Simulation results suggest that under inflation targeting inflation turns out to be more volatile than under the peg in the case of Latvia. Additional concern for output stabilisation accounts for lower inflation variability while it is still higher than under existing exchange rate regime with  $\pm 1\%$  fluctuation bands. The model results therefore support the existing exchange rate policy.

**Key words:** DSGE, small open economy, exchange rate policy, Bayesian estimation

**JEL:** C11, C3, C51, D58, E58, F41

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## INTRODUCTION

As of February 1994, the Bank of Latvia pegged the lats to the SDR basket of currencies and switched the peg to the euro in January 2005 setting the margins of the passive intervention target zone in both regimes at  $\pm 1\%$ . Latvia is participating in the exchange rate mechanism ERM II. While a standard ERM II requirement stipulates that a country should keep its exchange rate against the euro within a corridor of  $\pm 15\%$ , Latvia has unilaterally committed to limit the movements of the nominal exchange rate against the euro within the band of  $\pm 1\%$  around the central parity. Meeting the ERM II requirement alongside with the other Maastricht convergence criteria are the prerequisites to enter the EMU.

Some proponents of adopting the  $\pm 15\%$  fluctuation band argue that this offers an opportunity to conduct a relatively more independent monetary policy and direct inflation targeting could be a useful strategy for Latvia. Among other things, it also allows to bring down inflation through nominal appreciation. This naturally leads to the following questions. Will widening of the exchange rate target zone and carrying out monetary policy with elements of inflation targeting help to curb inflation in Latvia up to the level necessary to satisfy the Maastricht criterion? In what macroeconomic consequences such changes in the current monetary policy can result?

Over recent years, many central banks of industrial countries have begun to pursue inflation targeting as a monetary policy framework. As the experience of using inflation targeting for curbing inflation appeared to be quite successful in the countries which first applied this policy – New Zealand, Canada, and the United Kingdom – many other developed countries have adopted this experience despite the fact that inflation was relatively low in these countries.

Inflation targeting central banks usually adhere to the policy of a floating exchange rate. It is suggested that the floating exchange rate policy provides a degree of insulation against foreign monetary shocks and acts as a "shock absorber" which helps to stabilise the domestic economy in the face of the foreign monetary shocks.

However, a number of empirical studies suggest that the use of inflation targeting as monetary policy faces some difficulties. First, the neglecting of the exchange rate target for small open economies may lead to high exchange rate volatility and a strong impact on firms' profitability. Higher pass-through also means that domestic prices react strongly to exchange rate fluctuations. Second, investments financed by external borrowing are very vulnerable to large negative changes in capital inflows (the so called "sudden stops"). If foreign currency borrowing is relatively important in the balance sheets of financial institutions, production firms and the government, the large depreciation following a sudden stop under a floating exchange rate regime can lead to widespread bankruptcies. Third, the main advantage of a floating exchange rate regime – the ability to tailor monetary policy to the domestic economy and the domestic business cycle – is largely lost if the monetary authority enjoys little credibility. Changes in the interest rate will not be effective in influencing firms' pricing decision to meet the inflation target if firms do not believe the central bank will stick to the announced policy, and will falter in the face of output fluctuations. Imperfect credibility may then require large swings in interest rates for the central bank to achieve the inflation target. It will also force the central bank to adhere strictly to the inflation target, so as not to lose any credibility gained.

To evaluate different exchange rate policies for Latvia, we use a small open economy DSGE model following Lubik and Schorfheide (2007) using Bayesian methods and compare the simulation results under various policy rules. Results obtained from simulations of different exchange rate regimes and monetary policy rules provide some evidence that a fixed exchange rate regime ensures the lowest inflation variability. The model results are therefore in favour of the exchange rate policy currently pursued by the Bank of Latvia. Any changes in the policy would bring unfavourable consequences in terms of macro indicators and loss of credibility.

The paper is organised as follows. Section 1 presents a model framework which we proceed to estimate. Section 2 outlines the estimation strategy and the data. Section 3 contains our empirical results. Responses to shocks are covered in Section 4, while the final section concludes.

## 1. MODEL SETUP

The world economy is modelled as a continuum of small open economies represented by the unit interval. The performance of each economy does not have any impact on the rest of the world. Economies face imperfectly correlated productivity shocks while sharing identical preferences, technology, and market structure.

Since the main focus in the model is put on the behaviour of the single economy and its interaction with the rest of the world, and for the sake of notational simplicity, superscript  $i$  is omitted when referring to the small open economy being modelled. Variables with an  $i \in [0, 1]$  subscript refer to economy  $i$  as one among the continuum of economies constituting the world economy. Variables denoted by \* stand for the world economy as a whole.

### 1.1 Households

A representative household of a small open economy maximises its utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t / A_t, N_t) \quad (1)$$

where  $N_t$  denotes hours worked,  $A_t$  is a world technology process, and  $C_t$  is a composite consumption index defined as

$$C_t \equiv \left[ (1 - \alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (2).$$

$C_{H,t}$ , in its turn, is an index of consumption of domestic goods represented by the CES function

$$C_{H,t} \equiv \left( \int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where  $j \in [0, 1]$  denotes a differentiated good on the unit interval.  $C_{F,t}$  is an index of imported goods defined by

$$C_{F,t} \equiv \left( \int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

where  $C_{i,t}$  stands for an index of goods imported from country  $i$  and consumed by domestic households. As in the case of consumption of domestic goods, the index of imports is given by the CES function

$$C_{i,t} \equiv \left( \int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Parameter  $\varepsilon > 1$  implies the elasticity of substitution between goods produced within a specific country.  $\alpha \in [0, 1]$  measures a degree of openness which is commonly defined as the share of imports in GDP. Parameter  $\eta > 0$  denotes the substitutability between domestic and foreign goods from the standpoint of the domestic consumer, while  $\gamma$  denotes the substitutability between goods imported from different markets.

The household maximises its utility defined in (1) subject to a budget constraint

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)djdi + D_t \leq D_{t-1}R_t + W_tN_t + T_t \quad (3)$$

for  $t = 0, 1, 2, \dots$  where  $P_{H,t}(j)$  is the price of differentiated domestic good  $j$  and  $P_{i,t}(j)$  is the price of differentiated good  $j$  imported from country  $i$ .  $R_t$  is return on financial investment  $D_{t-1}$  held at the end of period  $t - 1$  (including shares in firms). Finally,  $W_t$  stands for the nominal wage, and  $T_t$  denotes lump-sum transfers (taxes).

## 1.2 Identities Between Inflation, Exchange Rates and Terms of Trade

Next, several identities linking inflation, exchange rates and terms of trade are defined. Bilateral terms of trade between the domestic economy and country  $i$  is given by

$$S_{i,t} = \frac{P_{H,t}}{P_{i,t}}$$

which is nothing but the price of home goods in terms of country  $i$ 's goods. Consequently, the effective terms of trade are defined as

$$S_t \equiv \frac{P_{H,t}}{P_{F,t}} = \left( \int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}.$$

Log-linearisation around the symmetric steady state gives

$$\pi_t = \pi_{H,t} - \alpha \Delta S_t \quad (4)$$

where  $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$  and lowercase letters stand for deviations from the steady state of the respective variables. Equation (4) implies that the inflation difference is proportional to the percent change in the terms of trade where the coefficient of proportionality is captured by the degree of openness  $\alpha$ .

Furthermore, it is assumed that the law of one price holds at a product level both for import and export prices, implying  $P_{i,t}(j) = \varepsilon_{i,t} P_{i,t}^i(j)$  for all  $i, j \in [0, 1]$ .  $\varepsilon_{i,t}$  is the bilateral nominal exchange rate, i.e., the price of country  $i$ 's currency in terms of the domestic currency, whereas  $P_{i,t}^i(j)$  is the price of country  $i$ 's good  $j$  denominated in its own currency terms. Applying the law-of-one-price assumption to the definition of  $P_{i,t}$  results in  $P_{i,t} = \varepsilon_{i,t} P_{i,t}^i$  where  $P_{i,t}^i \equiv \left( \int_0^1 P_{i,t}^i(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$  stands for country  $i$ 's domestic price index.

Next, for the purpose of exchange rate policy analysis, the nominal exchange rate  $e_t$  is introduced in the CPI inflation equation under the assumption that relative PPP holds. To derive this relationship, we express  $p_{F,t}$  from the terms of trade equation  $s_t = p_{H,t} - p_{F,t}$  and plug into  $p_{F,t} = e_t + p_t^*$  to obtain

$$p_{H,t} - s_t = e_t + p_t^* \quad (5).$$

Inserting differences

$$\pi_{H,t} - \Delta s_t = \Delta e_t + \pi_t^*$$

and using the expression for domestic inflation yields

$$\pi_t + \alpha \Delta s_t - \Delta s_t = \Delta e_t + \pi_t^*$$

or rearranging

$$\pi_t = \Delta e_t + (1 - \alpha) \Delta s_t + \pi_t^*.$$

Using the international risk sharing condition we obtain

$$\tilde{c}_t = \tilde{c}_t^* - \left( \frac{1 - \alpha}{\sigma} \right) s_t \quad (6)$$

where  $\tilde{c}_t^*$  stands for stationary log world consumption. This equation relates domestic consumption with world consumption and terms of trade.

### 1.3 Firms

The domestic economy is populated by a continuum of firms  $j \in [0, 1]$ , where each one produces a differentiated good using the same technology, represented by the production function

$$Y_t(j) = A_t N_t(j)$$

where  $A_t$  is the level of technology and  $a_t \equiv \log A_t$  is described by the AR(1) process  $a_t = \rho_a a_{t-1} + v_t$ .

All firms face identical demand curves and take the aggregate price level and aggregate consumption index exogenously. Following the price setting mechanism proposed by Calvo (1983), each firm may change its price with probability  $1 - \theta$  every period, irrespective of the last time of adjustment. Thus, each period a fraction  $1 - \theta$  of firms reset their prices, whereas the rest  $\theta$  keep their prices unchanged. In this way,  $\theta$  represents price stickiness.

Given that all firms resetting prices will choose the same price  $\bar{P}_{H,t}$ , the aggregate price level takes the form

$$P_{H,t} = \left[ \theta (P_{H,t-1})^{1-\varepsilon} + (1 - \theta) (\bar{P}_{H,t})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

Assuming a steady state with zero inflation  $\bar{P}_{H,t} = P_{H,t-1} = P_{H,t}$  for all  $t$ , log-linearisation of the last expression around the steady state results in

$$\pi_{H,t} = (1 - \theta)(\bar{P}_{H,t} - p_{H,t-1}) \quad (7).$$

Equation (7) implies that inflation results from firms re-optimising their price each period so that it differs from the average  $t - 1$  period price in the economy. Therefore, to follow the inflation dynamics in the course of time, the next step is to clarify the factors underlying firms' price setting decisions.

A firm re-optimising in period  $t$  will choose the price  $\bar{P}_{H,t}$  to maximise the present market value of its profits generated while the price remains effective

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( \bar{P}_{H,t} Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\} = 0 \quad (8)$$

subject to the set of demand constraints

$$Y_{t+k|t} = \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left( C_{H,t+k} + \int_0^1 C_{H,t+k}^i di \right) = \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \hat{C}_{H,t+k} \equiv Y_{t+k}^d(\bar{P}_{H,t}) \quad (9)$$

for  $k = 0, 1, 2, \dots$  where  $Q_{t,t+k} \equiv \beta^k (\tilde{C}_{t+k} / \tilde{C}_t)^{-\sigma} (A_t / A_{t+k})(P_t / P_{t+k})$  is the stochastic discount factor for nominal payoffs,  $\Psi_t(\cdot)$  is the cost function, and  $Y_{t+k|t}$  denotes the  $t + k$  period output of a firm that last reset its price in period  $t$ .

Solving the problem (9) and log-linearising results in

$$\bar{p}_{H,t} - p_{H,t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \hat{mc}_{t+k|t} + (p_{H,t+k} - p_{H,t-1}) \right\} \quad (10)$$

where  $\hat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$  stands for the log deviation of marginal cost from its steady state value  $mc$ .

## 1.4 Equilibrium

### 1.4.1 The demand side

Goods market clearing in the domestic economy requires

$$\frac{Y_t(j)}{A_t} = \frac{C_{H,t}(j)}{A_t} + \int_0^1 \frac{C_{H,t}^i(j)}{A_t} di$$

for all  $j \in [0, 1]$  and all  $t$ , where  $C_{H,t}^i(j)$  stands for country  $i$ 's demand for the domestically produced good  $j$ .

### 1.4.2 The supply side

Let  $Y_t \equiv \left[ \int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$  represent an index for aggregate domestic output. One can derive a production function linking the aggregate domestic demand with aggregate employment. Market clearing in the labour market requires  $N_t = \int_0^1 N_t(j) dj$ .

Expressing labour demand from the firm's production function as  $N_t(j) = Y_t(j) / A_t$  and plugging into the labour market clearing condition yields

$$N_t = \int_0^1 \frac{Y_t(j)}{A_t} dj = \int_0^1 \frac{Y_t(j) Y_t}{Y_t A_t} dj = \frac{Y_t}{A_t} \int_0^1 \frac{Y_t(j)}{Y_t} dj.$$

Standard derivations yield domestic inflation as a function of deviations of marginal cost from its steady state value

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \lambda \hat{m}c_t \quad (11)$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

Equation (11) implies that inflation for domestically produced goods is not affected by parameters referring to the open economy. Conversely, real marginal cost as a function of domestic output in the open economy does differ from the closed economy case which results from the wedge between output and consumption, and between domestic and consumer prices.

After some manipulations, we come up with the real marginal cost as a function of domestic output and world output

$$\hat{m}c_t = (\sigma_\alpha + \varphi) \tilde{y}_t + (\sigma - \sigma_\alpha) \tilde{y}_t^* \quad (12).$$

### 1.5 Monetary Policy

Monetary policy is defined by an interest rate rule in a way that the central bank sets its policy rate to adjust for movements in CPI inflation, output, and nominal exchange rate depreciation  $\Delta e_t$

$$r_t = \rho_R r_{t-1} + (1 - \rho_R) [\psi_1 \pi_t + \psi_2 \tilde{y}_t + \psi_3 \Delta e_t] + \varepsilon_t^R$$

where the policy coefficients  $\psi_1, \psi_2, \psi_3 \geq 0$ , and  $\varepsilon_t^R$  stands for an exogenous policy shock. To describe the persistence in nominal interest rates, a smoothing term given by  $0 < \rho_R < 1$  is incorporated in the policy rule.

## 1.6 A Simplified Version

In the paper, we estimate a simplified version of Gali and Monacelli (2005) model presented above where  $\varphi = 0$ ,  $\eta = 1$ ,  $\gamma = 1$ , and  $1/\sigma = \tau$ . The open economy model consists of a forward-looking IS equation and the New Keynesian Phillips curve. Monetary policy is defined by an interest rate rule, whereas the exchange rate is introduced via the CPI equation given that PPP holds. Foreign output, foreign inflation, and changes in terms of trade are defined by autoregressive processes.

Finally, we provide a brief overview of the key final log-linearised equations of the model which we will use for estimation:

$$\tilde{y}_t = E_t \{\tilde{y}_{t+1}\} - (\tau + \alpha(1-\tau)(2-\alpha))(r_t - E_t \{\pi_{t+1} + \alpha\Delta s_{t+1}\} - \rho_z z_t) + \alpha(2-\alpha) \left( \frac{1-\tau}{\tau} \right) E_t \{\Delta \tilde{y}_{t+1}^*\} \quad (13)$$

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \alpha \beta E_t \{\Delta s_{t+1}\} - \alpha \Delta s_t + \frac{\lambda}{\tau + \alpha(2-\alpha)(1-\tau)} (\tilde{y}_t - \tilde{y}_t^n) \quad (14)$$

$$\pi_t = \Delta e_t + (1-\alpha)\Delta s_t + \pi_t^* \quad (15)$$

$$r_t = \rho_R r_{t-1} + (1-\rho_R)[\psi_1 \pi_t + \psi_2 \tilde{y}_t + \psi_3 \Delta e_t] + \varepsilon_t^R \quad (16)$$

$$\Delta s_t = \rho_s \Delta s_{t-1} + \varepsilon_t^s \quad (17)$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_t^{y^*} \quad (18)$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_t^{\pi^*} \quad (19)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \quad (20)$$

$$\text{where } \tilde{y}_t^n \equiv -\frac{\alpha(2-\alpha)(1-\tau)}{\tau} \tilde{y}_t^* \text{ and } z_t \equiv \Delta a_t.$$

Equation (13) is the open economy IS curve implying that output depends on the expectations of future output both at home and abroad, the real interest rate, the expected changes in the terms of trade, and technology growth. Equation (14) represents the New Keynesian open economy Phillips curve. Movements in the output gap affect inflation as they are associated with changes in real marginal costs, whereas the parameter  $\lambda$  affects the slope of the Phillips curve and is a function of other deeper parameters, but here it is taken to be structural. Changes in the terms of trade enter the Phillips curve reflecting the fact that some consumer goods are imported. Equation (15) is a PPP version. The monetary policy in (16) is described by an interest rate rule, where the central bank adjusts its instrument in response to deviations of CPI inflation from the target and output from its potential level, as well as to changes in the exchange rate. A smoothing coefficient is introduced that reflects the degree of persistence in the policy instrument. The rest of equations refer to exogenous terms of trade, foreign output, inflation and technology respectively. All follow a first-order autoregressive process.

## 2. MODEL ESTIMATION

### 2.1 Data Description

The Bayesian approach pursued in this paper has three main characteristics. First, unlike GMM (generalised method of moments) estimation, the Bayesian analysis is system-based and fits the solved DSGE model to a vector of aggregate time series. Second, the estimation is based on the likelihood function generated by the DSGE model rather than, for instance, the discrepancy between DSGE model impulse response functions and identified VAR (vector autoregression) impulse responses as in Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005). Third, prior distributions can be used to incorporate additional information into the parameter estimation.

We use observations on real output growth, inflation, nominal interest rates, exchange rate changes, and terms of trade changes in our empirical analysis. All data are at quarterly frequencies for the period from the second quarter of 1998 to the second quarter of 2007. Output growth rates are computed as log differences of GDP and scaled by 100 to convert them into quarter-on-quarter percentage changes. Inflation rates are defined as log differences of the consumer price indices CPI and multiplied by 400 to obtain annualised percentage changes. The terms of trade, defined as the relative price of exports in terms of imports, is converted in log differences (scaled by 100) to obtain percentage changes. We use the overnight money market rate as a policy rate. For exchange rate series, we take the average of commercial banks' bid and ask rates of the lats against the SDR until December 2004 and those against the euro afterwards. We use log differences (scaled by 100) of exchange rates to obtain percentage deviation from the parity level to the SDR and euro in the respective periods. Both overnight rates and exchange rates are averaged over the respective quarter. GDP, CPI, export and import price indices are seasonally adjusted. All series are demeaned prior to estimation.

### 2.2 Choice of Priors

Table 1 provides information about the priors for Latvia. We choose priors for the structural parameters to be estimated based on several considerations. Prior distributions are assumed to be independent. We use fairly loose priors for the parameters of the policy rule. The priors for  $\psi_1$  and  $\psi_2$  are centred at the values commonly associated with the Taylor-rule. Smets and Wouters (2004), for example, reported posterior means for inflation and output gap coefficients at the values of 1.5 and 0.06, respectively, in their estimated DSGE framework for the euro area. The prior mean of the exchange rate coefficient is set at the high value of 400 to ensure the fixed exchange rate regime. Our rule also allows for interest rate smoothing with a prior mean of 0.2 with a standard deviation of 0.1. The model is parameterised in terms of the steady state real interest rate  $r$  rather than the discount factor  $\beta$ .  $r$  is annualised such that  $\beta = \exp[-r/400]$ . Its mean is chosen to be 2.5%. The prior for the slope coefficient  $\lambda$  in the Phillips curve is consistent with values reported in the literature (see, for instance, Lubik and Schorfheide, 2007; Rotemberg and Woodford, 1997; Gali and Gertler, 1999; Sbordone, 2002). Its mean is set at 0.5, but we allow it to vary widely. The prior for the preference parameter  $\alpha$ , the import share, is centred at 0.4 which corresponds to the average value over the period.

We specify priors for the exogenous shocks following Lubik and Schorfheide (2007). To define the priors for the exogenous shocks for world inflation, we fit an AR process to the EU CPI quarterly inflation and center a prior for  $\rho_{\pi^*}$  at 0.5 with  $\sigma_{\pi^*}$  at 0.25.

Priors for the rest-of-the-world output shock  $y_t^*$  are obtained by estimating an AR(1) for the log of ratio of the EU GDP to domestic GDP. We set the prior for  $\rho_{y^*}$  at 0.99 and use 0.75 to center the prior of the standard deviation.

To specify the prior for the technology process, we fit AR(1) to the domestic output growth rate. The point estimates for the autoregression coefficient and standard deviation are 0.12 and 1.11 respectively. We thus choose 0.1 as the prior mean for  $\rho_z$  and 1.0 for  $\sigma_z$ .

For the terms of trade changes, the estimated autoregressive coefficient is 0.11, and the standard error of regression is 2.14; thus we choose the prior mean of 0.1 for  $\rho_s$  and 2.0 for  $\sigma_s$ .

We choose 0.45 as a mean for the  $\sigma_r$  prior which corresponds to the standard deviation of AR(1) for the interest rate.

### 3. RESULTS

The Bayesian estimates of the structural parameters for Latvia are shown in Table 1. In addition to 90% posterior probability intervals, we report posterior means as point estimates.<sup>1</sup> First, let us look at the results obtained for 60 000 iterations. Compared to countries pursuing inflation targeting and therefore having a large inflation coefficient in Taylor rule, the low value of  $\psi_1$  for Latvia is consistent with the Bank of Latvia monetary policy. A low output gap coefficient ( $\psi_2 = 0.016$ ) implies no primary concern for output deviations, whereas a high value of the exchange rate parameter ( $\psi_3 = 44.8$ ) confirms the fixed exchange rate policy pursued by the Bank of Latvia. There is also a very high degree of interest-smoothing with an estimate of  $\rho_r = 0.9$ .

The estimates of the structural parameters fall within plausible ranges. The preference parameter  $\alpha$  is estimated to mimic the observable average Latvian import share over the last years. The estimate of the Phillips-curve parameter  $\lambda$  is slightly higher than its prior reflecting the fact that domestic firms strongly react to output deviations in their optimal price setting behaviour. Intertemporal substitution elasticity  $\tau$  appears surprisingly low, indicating that consumers are less willing than expected to change their consumption decisions in response to interest rate shocks. The estimates of the stochastic processes reflect the substantial degree of persistence found in the data, most of which is captured by the high degree of autocorrelation in technology growth ( $\rho_z = 0.61$ ) and the foreign demand shock ( $\rho_{y^*} = 0.95$ ).

Table 1

#### Prior distributions and posterior estimation results for Latvia

Prior					Posterior (60 000 iterations)			Posterior (1 000 000 iterations)		
Name	Domain	Density	Mean	St. dev.	Mean	90% interval		Mean	90% interval	
$\psi_1$	$\mathbb{R}^+$	Gamma	2.00	0.50	0.515	0.251	0.712	1.256	0.762	1.735
$\psi_2$	$\mathbb{R}^+$	Gamma	0.05	0.13	0.016	0.000	0.032	0.034	0.000	0.108
$\psi_3$	$\mathbb{R}^+$	Gamma	400	100	44.80	44.78	44.82	44.81	44.78	44.86
$\rho_r$	[0, 1)	Beta	0.20	0.10	0.896	0.896	0.896	0.894	0.894	0.894
$\alpha$	[0, 1)	Beta	0.40	0.20	0.627	0.623	0.638	0.665	0.563	0.854
$\Gamma$	$\mathbb{R}^+$	Gamma	2.50	0.50	2.292	1.928	2.686	2.515	1.585	3.215
$\lambda$	$\mathbb{R}^+$	Gamma	0.50	0.25	0.618	0.607	0.625	1.598	1.338	1.796
$\tau$	[0, 1)	Gamma	0.20	0.10	0.153	0.147	0.156	0.127	0.073	0.194
$\rho_s$	[0, 1)	Beta	0.10	0.05	0.137	0.134	0.141	0.356	0.295	0.422
$\rho_z$	[0, 1)	Beta	0.10	0.05	0.606	0.606	0.606	0.605	0.605	0.605
$\rho_{y^*}$	[0, 1)	Beta	0.99	0.05	0.954	0.922	0.993	0.986	0.961	1.000
$\rho_{\pi^*}$	[0, 1)	Beta	0.50	0.20	0.422	0.420	0.424	0.135	0.061	0.198
$\sigma_r$	$\mathbb{R}^+$	InvGamma	0.45	4.00	0.712	0.705	0.720	1.948	1.755	2.141
$\sigma_s$	$\mathbb{R}^+$	InvGamma	2.00	4.00	1.594	1.472	1.735	2.123	1.835	2.613
$\sigma_z$	$\mathbb{R}^+$	InvGamma	1.00	4.00	1.275	1.232	1.314	1.722	1.561	1.873
$\sigma_{y^*}$	$\mathbb{R}^+$	InvGamma	0.75	4.00	0.924	0.903	0.949	0.578	0.200	0.939
$\sigma_{\pi^*}$	$\mathbb{R}^+$	InvGamma	0.25	4.00	0.319	0.312	0.327	1.268	1.129	1.397

The results on prior and posterior distributions of the model parameters are reported in Appendix 1. Most of posterior distributions are shifted with respect to priors and highly concentrated around their mean values. This implies that overall the data are

<sup>1</sup> We construct the posteriors using the Metropolis-Hastings algorithm with a Markov chain 60 000 and 1 000 000 observations long. All estimation was conducted using Dynare 3, in Matlab R2008a.

informative and the parameter estimates are close to their true values. The only exception where the data provide little information is the interest rate.

To evaluate different exchange rate policies for Latvia, we simulate the model using different policy parameters and compare the results under various policy rules. First we derive results by applying coefficients estimated from the data, i.e., we use  $\psi_1 = 0.515$ ,  $\psi_2 = 0.016$ ,  $\psi_3 = 44.801$  and refer to this case as a benchmark model. Table 2 provides the results. Under these parameter values, the exchange rate appears to fit into the existing regime of  $\pm 1\%$  band with 99% probability. Furthermore, we simulate a change in the monetary policy by allowing wider exchange rate bands. We consider three different values for  $\psi_3$  – 2.0, 1.0, and 0.6 while leaving the rest of the estimated coefficients unchanged. At  $\psi_3 = 2.0$ , the exchange rate volatility increases 5.9 times, inflation variability amplifies, while interest rate fluctuations become less pronounced which is consistent with the diminishing role of the interest rate in exchange rate stabilisation. Relaxing the bands even further ( $\psi_3 = 1.0$ ), the effect on inflation variation becomes more pronounced. Surprisingly, under wider exchange rate bands output fluctuations amplify, slightly though. This implies that the exchange rate does not serve as a shock absorber.

Finally, we simulate the model for the latter case ( $\psi_3 = 0.6$ ) but allow for different inflation and output gap targeting policies. Column 2 of Table 3 provides results for the case where the inflation coefficient assumes the value commonly associated with the Taylor rule ( $\psi_1 = 1.5$ ), whereas the output gap coefficient is left as derived from the data ( $\psi_2 = 0.016$ ). As expected, inflation targeting brings down inflation fluctuations compared to cases where the central bank demonstrates no concern about the price changes. What is surprising, though: under inflation targeting inflation turns out to be more volatile than under the peg. Under an even tighter inflation targeting regime ( $\psi_1 = 2.0$ ), inflation fluctuations decrease (see Column 3 of Table 3) while still exceeding the respective value of the benchmark model. In the third case, we leave inflation targeting as in the previous case while allowing the central bank to also target the output gap. Additional concern for output stabilisation accounts for lower inflation variability compared to the two other cases covered before while it is still higher than under the existing exchange rate regime with  $\pm 1\%$  fluctuation bands.

Table 2

**Standard deviations for the benchmark model and various exchange rate regimes**

Variable	Benchmark model	$\psi_1 = 0.515, \psi_2 = 0.016$		
	$\psi_1 = 0.515, \psi_2 = 0.016,$ $\psi_3 = 44.801$	$\psi_3 = 2.0$	$\psi_3 = 1.0$	$\psi_3 = 0.6$
$z$	1.605	1.619	1.626	1.630
$\Delta e$	0.329	1.944	2.880	3.689
$\Delta s$	1.596	1.626	1.597	1.604
$\pi$	2.275	7.320	11.104	14.403
$r$	5.518	1.932	1.736	1.701
$y$	5.149	5.262	5.617	5.885

Table 3

Standard deviations for  $\psi_3 = 0.6$  and various inflation and output gap targeting regimes

Variable	$\psi_1 = 1.5, \psi_2 = 0.016$	$\psi_1 = 2, \psi_2 = 0.016$	$\psi_1 = 2, \psi_2 = 0.6$
z	1.613	1.609	1.597
$\Delta e$	2.282	2.007	1.819
$\Delta s$	1.620	1.597	1.595
$\pi$	8.403	7.156	6.182
r	1.824	1.898	1.937
y	5.281	5.224	5.095

Next, we estimate the initial model with the same priors, but also include the Bayesian estimation of impulse response functions in exogenous shocks. To construct the posteriors using the Metropolis-Hastings algorithm with a Markov chain, we increase the number of iterations to 1 000 000.

The results of Bayesian posteriors along with the initial model estimates are reported in the last three columns of Table 1. The posterior means of the structural parameters in both estimation rounds are rather similar. There is quite robust evidence of the fixed exchange rate policy pursued by the Bank of Latvia ( $\psi_3 = 44.8$ ), while the rather low value of  $\psi_2$  implies no primary concern about output deviations from the policy rule.

As before, we simulate the model using different policy parameters and compare the results (see Tables 4 and 5). Policy implications are similar to the ones obtained for the first simulation round. At the extreme case where the central bank pursues a close to free float policy ( $\psi_3 = 0.6$ ) implying exchange rate fluctuations in the  $\pm 15\%$  range with 99% probability, inflation variability exceeds fivefold its benchmark level under the fixed exchange rate regime with  $\pm 1\%$  bands. Again, the exchange rate does not appear to absorb output fluctuations.

Table 4

## Standard deviations for the benchmark model and various exchange rate regimes

Variable	Benchmark model	$\psi_1 = 1.256, \psi_2 = 0.034$		
	$\psi_1 = 1.256, \psi_2 = 0.034, \psi_3 = 44.814$	$\psi_3 = 2.0$	$\psi_3 = 1.0$	$\psi_3 = 0.6$
z	1.620	1.601	1.618	1.630
$\Delta e$	0.336	3.206	4.523	5.356
$\Delta s$	1.574	1.585	1.524	1.552
$\pi$	4.652	12.733	17.938	21.273
r	4.079	2.276	2.193	2.150
y	4.917	5.344	5.435	5.912

*Table 5***Standard deviations for  $\psi_3 = 0.6$  and various inflation and output gap targeting regimes**

Variable	$\psi_1 = 1.5, \psi_2 = 0.034$	$\psi_1 = 2, \psi_2 = 0.034$	$\psi_1 = 2.5, \psi_2 = 0.6$
$z$	1.634	1.606	1.625
$\Delta e$	4.701	4.177	3.321
$\Delta s$	1.544	1.588	1.545
$\pi$	18.467	15.950	12.249
$r$	2.186	2.233	2.410
$y$	5.453	5.416	5.179

Results obtained from simulations of various exchange rate regimes and monetary policy rules thus provide some evidence that under the peg inflation variability is lower compared to other policy rules. The model results are therefore in favour of the exchange rate policy currently pursued by the Bank of Latvia. Any changes in the policy would bring unfavourable consequences in terms of macro indicators and loss of credibility to the monetary authority.

#### 4. RESPONSES TO SHOCKS

In order to gauge the importance of the individual shocks, we estimate impulse response functions. The results for simulations with posterior mean parameters are reported in Appendix 2.

An improvement in the terms of trade raises output and inflation on impact via a nominal appreciation. This mechanism works in the following way. At the outset, the improvement of the terms of trade will increase the income of the domestic exporters, leading to activity and employment growth in the export industries. Since exporters are earning more foreign currency, they will increasingly bring it to the foreign exchange market. As foreign currency becomes abundant, appreciation pressures on the domestic currency would appear. Under the fixed exchange rate policy, the central bank will be required to intervene in the foreign exchange market to maintain the peg. Thus, the central bank will sustain the value of the domestic currency by purchasing foreign currency. This move will in turn increase the amount of domestic currency and credit available for business investment and expansion. As the impact of these central bank actions is equivalent to that of expansionary monetary policy, the response to the rise in export prices will lead to output growth and raise inflation through higher marginal costs.

Positive productivity shocks raise output and subsequently increase marginal cost of firms. Higher marginal costs are directly translated into domestic inflation which makes domestic goods less competitive and urges agents to shift their consumption to foreign substitutes. Lower demand for domestic goods results in depreciation pressures to the extent the central bank reacts to the downward pressure on the exchange rate by raising its policy rate.

As a result of the import price inflation shock, agents substitute consumption of imports with relatively more price-competitive domestic goods, thus raising the demand for domestic currency and causing exchange rate appreciation. The central bank reacts to movements in the exchange rate by subsequently relaxing monetary policy which has an expansionary effect on output. Output growth raises the marginal cost of firms which ultimately is reflected in higher inflation.

Bayesian estimates of impulse responses to shocks are reported in Appendix 4. The posterior distributions of the impulse responses are constructed by pulling parameters, together with the variances of the shocks, from the corresponding posterior distributions and for each set of draws generating an impulse response. Repeating this process many times generates posterior distributions of impulse responses.

The plots in Appendix 4 show 90% confidence intervals of impulse response distributions to the monetary, terms of trade, productivity, and foreign inflation shocks. The posterior distributions of the impulse responses look quite similar to the ones obtained from simulations with posterior mean parameters. The confidence intervals of the impulse responses appear to be quite narrow, indicating that the responses are statistically significant.

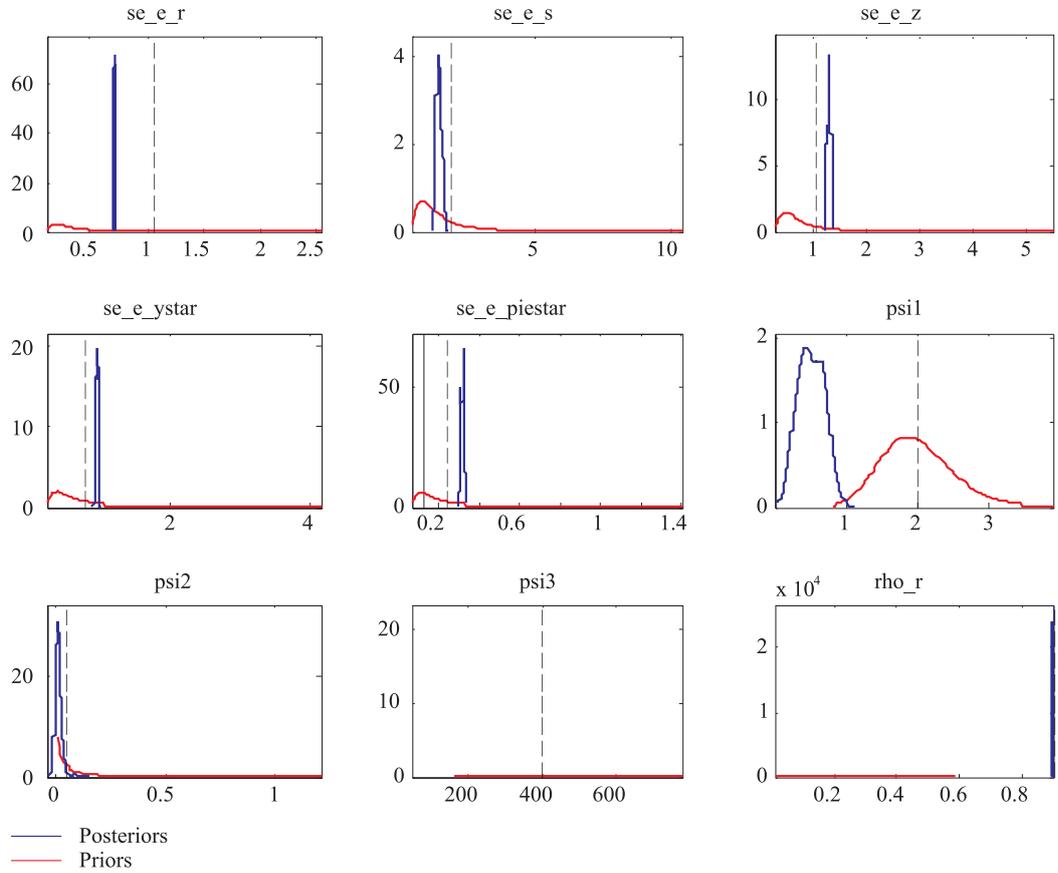
## CONCLUSIONS

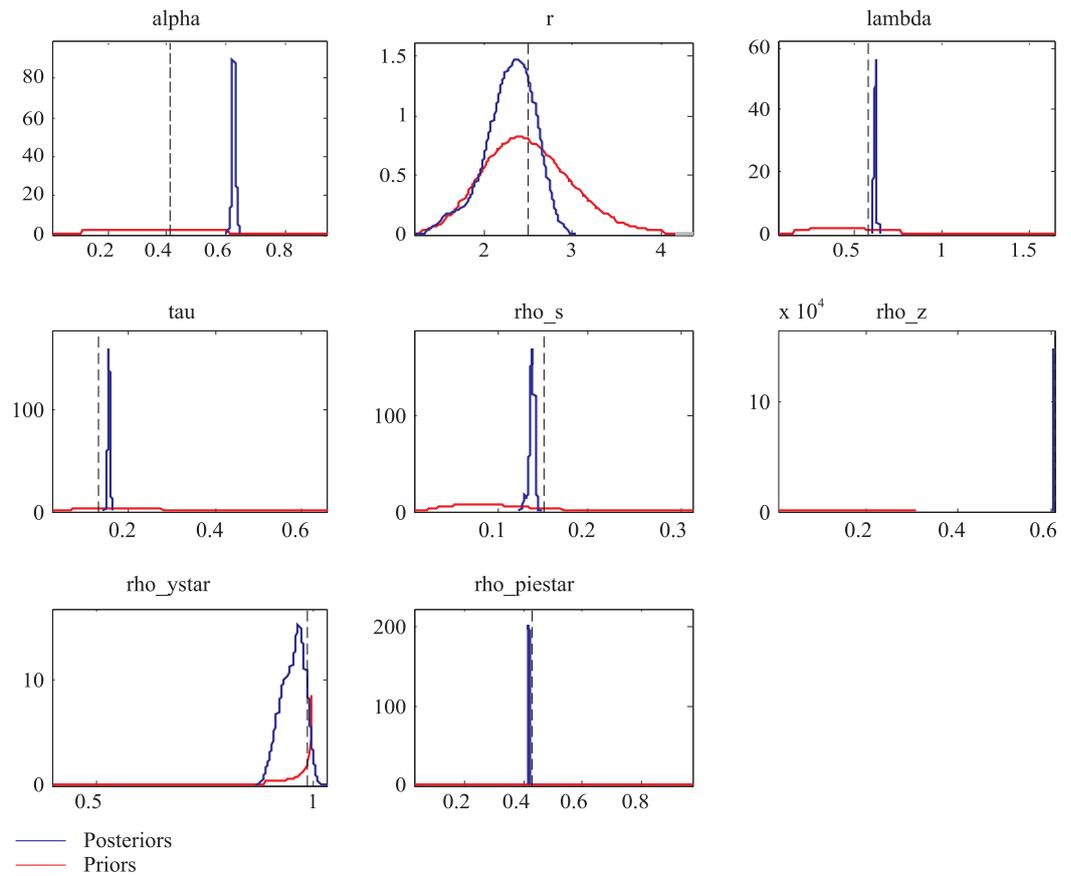
In this paper we estimate a small open economy DSGE model for Latvia, following Lubik and Schorfheide (2007) using Bayesian methods. The estimates of the structural parameters fall within plausible ranges. To evaluate different exchange rate policies for Latvia, we simulate the model using different policy parameters and compare the results under various policy rules. In extreme cases where the central bank pursues a close to free float policy, inflation fluctuations exceed fivefold their level under the currently pursued fixed exchange rate regime. Simulation results for the free float regime, allowing for different inflation and output gap targeting policies, suggest that inflation targeting brings down inflation fluctuations compared to cases where the central bank demonstrates no concern for the price changes. What is surprising, however, is that under inflation targeting inflation turns out to be more volatile than under the peg. Additional concern for output stabilisation accounts for lower inflation variability while it is still higher than under the existing exchange rate regime with  $\pm 1\%$  fluctuation bands. The model results therefore are in favour of the existing exchange rate policy. Any changes in the policy would bring unfavourable consequences in terms of macro indicators and loss of credibility to the monetary authority.

**APPENDICES**

*Appendix 1*

**Priors and posteriors**



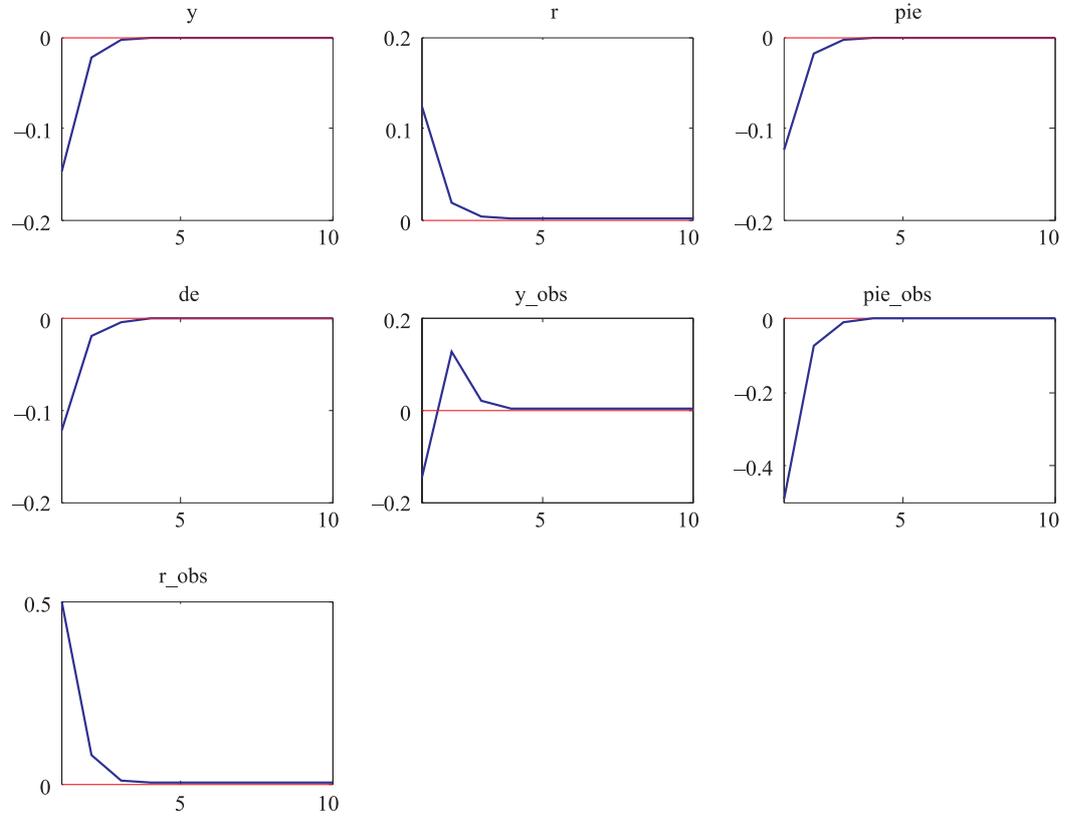


Notes:  $se\_e\_r$ ,  $se\_e\_s$ ,  $se\_e\_z$ ,  $se\_e\_ystar$ , and  $se\_e\_piestar$  are standard errors of the shocks  $\varepsilon_t^R$ ,  $\varepsilon_t^S$ ,  $\varepsilon_t^Z$ ,  $\varepsilon_t^{y^*}$ , and  $\varepsilon_t^{\pi^*}$  respectively.  $\psi_1$ ,  $\psi_2$ ,  $\psi_3$ ,  $\rho_r$ ,  $\rho_s$ ,  $\rho_z$ ,  $\rho_{y^*}$ ,  $\rho_{\pi^*}$ ,  $\alpha$ ,  $\lambda$ , and  $\tau$  respectively.  $r$  is the steady state real interest rate.

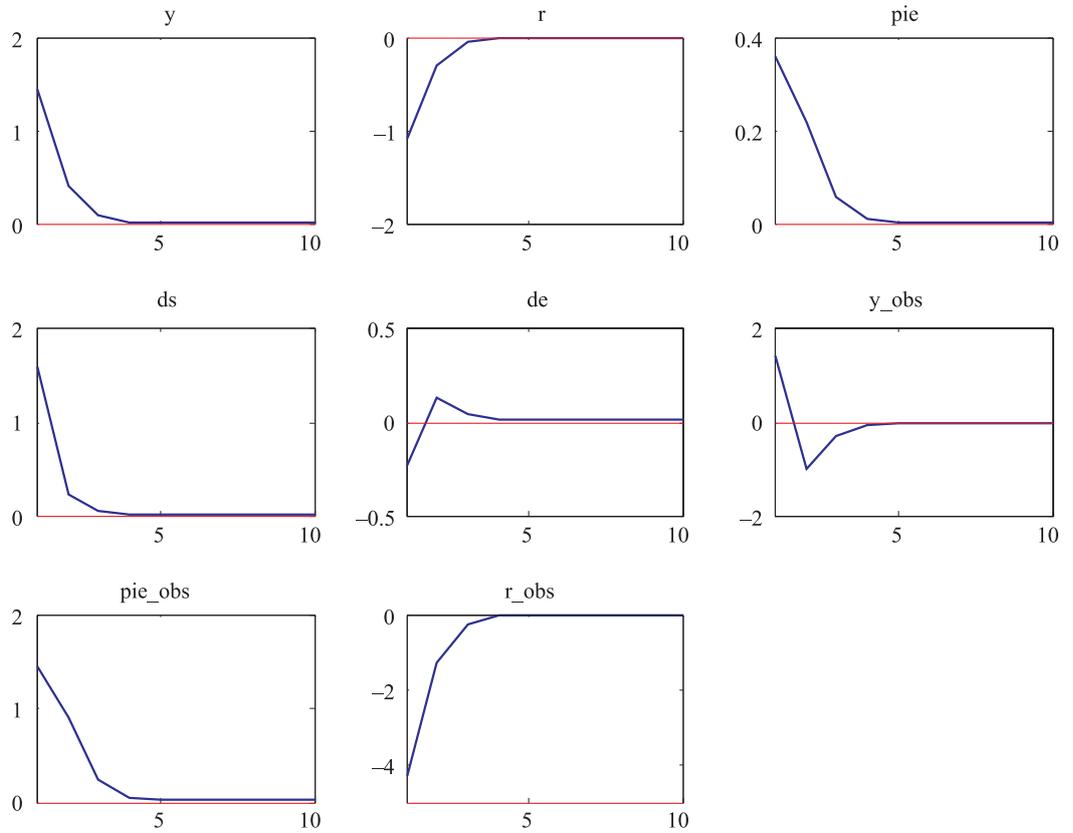
*Appendix 2*

**Impulse responses to shocks**

**Response to monetary shock  $\varepsilon^r$**

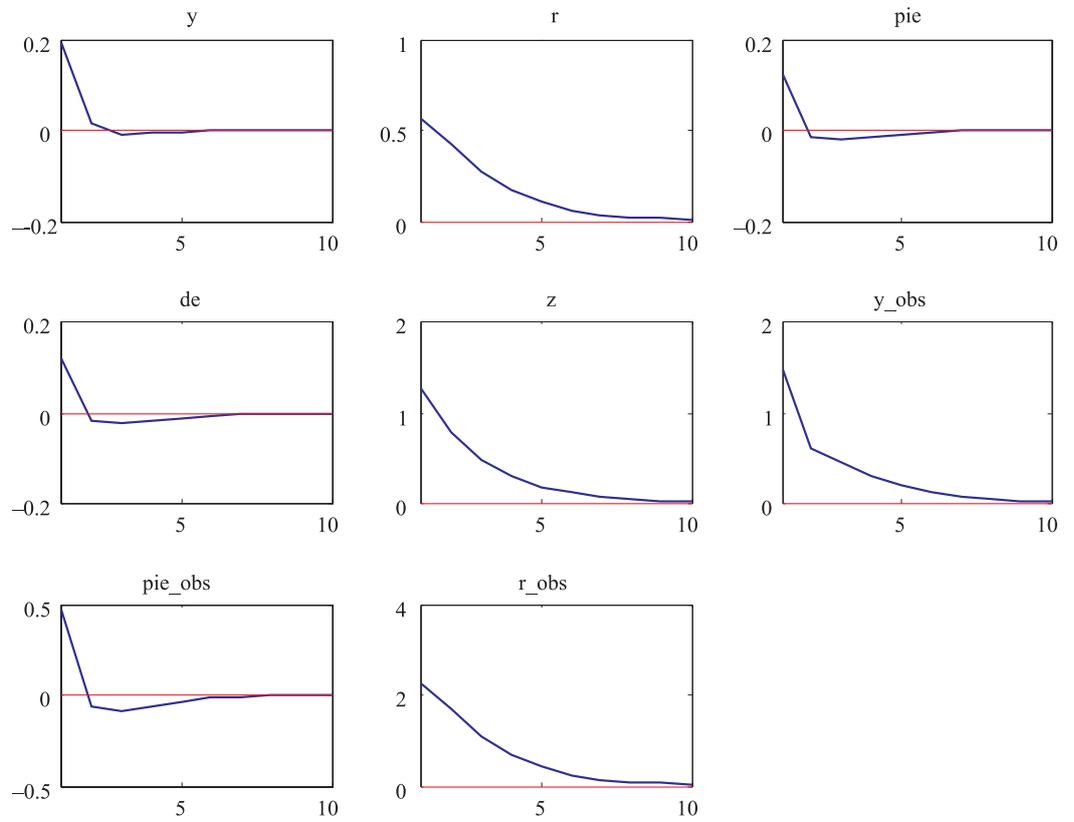


**Response to terms of trade shock  $\varepsilon^s$**

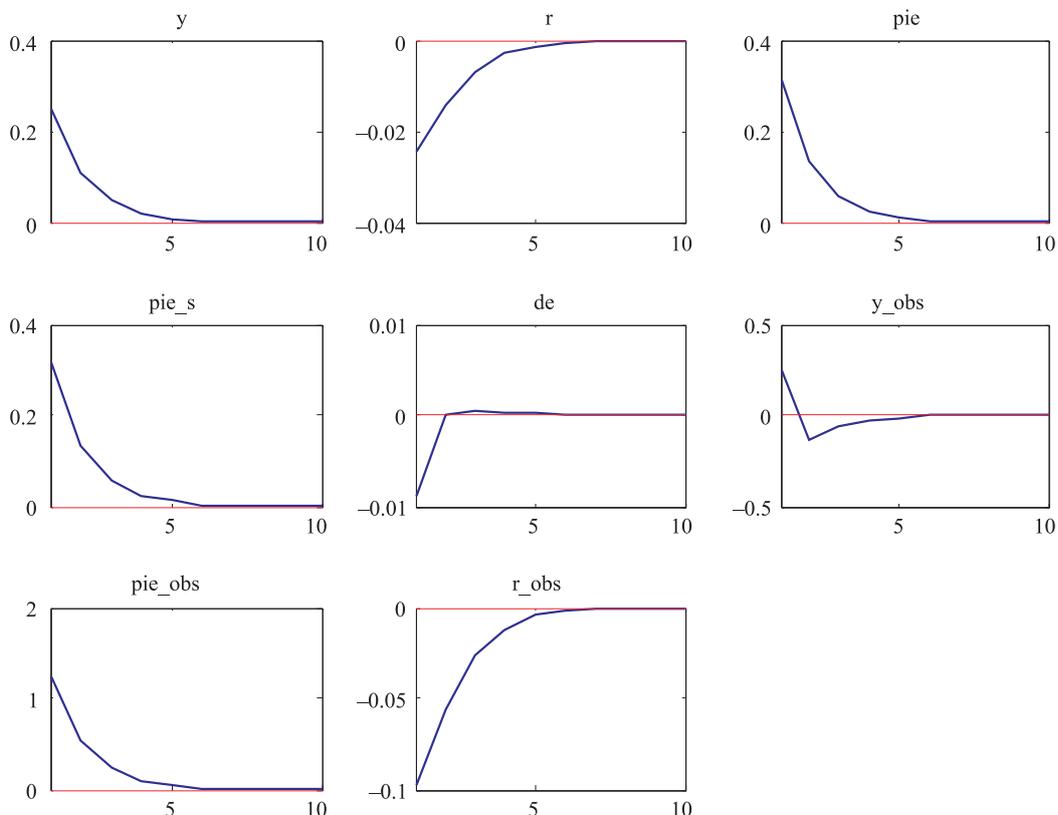


Notes:  $y$ ,  $r$ , and  $pie$  are output, interest rate and CPI inflation deviations from the steady state ( $\tilde{y}_t$ ,  $r_t$ , and  $\pi_t$ , respectively, in the model equations). The steady state is the mean value over the estimation period.  $de$  stands for the exchange rate percentage deviation from the parity level to SDR and EUR in the respective periods ( $\Delta e_t$ ).  $ds$  is the terms of trade change ( $\Delta s_t$ ).  $r$ ,  $pie$ ,  $de$ , and  $ds$  are at quarterly frequencies.  $pie\_obs$  and  $r\_obs$  are the respective  $pie$  and  $r$  in annual terms.  $y\_obs = y - y(-1) + z$ .

**Response to productivity shock  $\varepsilon^z$**



**Response to foreign inflation shock  $\varepsilon^{\pi^*}$**



Notes:  $y$ ,  $r$ ,  $pie$ , and  $pie_s$  are output, interest rate, CPI inflation, and foreign inflation deviations from the steady state ( $\tilde{y}_t$ ,  $r_t$ ,  $\pi_t$ , and  $\pi_t^*$  respectively, in the model equations). The steady state is the mean value over the estimation period.  $de$  stands for the exchange rate percentage deviation from the parity level to SDR and EUR in the respective periods ( $\Delta e_t$ ).  $ds$  is the terms of trade change ( $\Delta s_t$ ).  $z$  is the technology growth rate ( $z_t$ ).  $r$ ,  $pie$ ,  $de$ , and  $ds$  are at quarterly frequencies.  $pie\_obs$  and  $r\_obs$  are the respective  $pie$  and  $r$  in annual terms.  $y\_obs = y - y(-1) + z$ .

## Appendix 3

**Matrix of covariance of exogenous shocks**

Variables	$\varepsilon^r$	$\varepsilon^s$	$\varepsilon^{y^*}$	$\varepsilon^{\pi^*}$	$\varepsilon^z$
$\varepsilon^r$	0.5062	0	0	0	0
$\varepsilon^s$	0	2.5402	0	0	0
$\varepsilon^{y^*}$	0	0	0.8545	0	0
$\varepsilon^{\pi^*}$	0	0	0	0.1016	0
$\varepsilon^z$	0	0	0	0	1.6246

**Policy and transition functions**

		$\Delta e$	$y_{obs}$	$\pi_{obs}$	$r_{obs}$
$r$	(-1)	-0.156	-0.188	-0.625	0.624
$\pi^*$	(-1)	-0.012	0.332	1.642	-0.130
$z$	(-1)	0.056	0.696	0.223	1.054
$y$	(-1)	0	-1	0	0
$y^*$	(-1)	0	-4.560	0	0
$\Delta s$	(-1)	-0.020	0.123	0.123	-0.373
$\varepsilon^r$		-0.175	-0.210	-0.698	0.696
$\varepsilon^s$		-0.149	0.898	0.898	-2.726
$\varepsilon^{y^*}$		0	-4.780	0	0
$\varepsilon^{\pi^*}$		-0.028	0.787	3.888	-0.308
$\varepsilon^z$		0.092	1.148	0.367	1.739

**Moments of simulated variables**

Variable	Mean	St. dev.	Variance	Skewness	Kurtosis
$z$	0.031	1.555	2.418	0.014	-0.029
$\Delta e$	-0.002	0.324	0.105	0.066	-0.049
$\Delta s$	0.011	1.611	2.595	-0.018	-0.159
$\pi$	-0.005	0.574	0.330	-0.055	-0.123
$\pi_{obs}$	-0.021	2.297	5.276	-0.055	-0.123
$\pi^*$	-0.008	0.356	0.127	-0.023	-0.161
$r$	0.009	1.382	1.909	-0.047	0.006
$r_{obs}$	0.037	5.526	30.541	-0.047	0.006
$y$	-2.817	15.527	241.084	-0.238	0.661
$y_{obs}$	0.028	5.100	26.014	-0.059	-0.218
$y^*$	0.590	3.223	10.389	0.219	0.628

**Correlation of simulated variables**

Variable	z	$\Delta e$	$\Delta s$	$\pi$ obs	$\pi^*$	r obs	y obs	$y^*$
z	1.000	0.214	-0.013	0.151	0.069	0.553	0.343	-0.062
$\Delta e$	0.214	1.000	-0.649	-0.130	-0.026	0.521	-0.206	0.009
$\Delta s$	-0.013	-0.649	1.000	0.674	-0.010	-0.823	0.261	-0.016
$\pi$ obs	0.151	-0.130	0.674	1.000	0.595	-0.548	0.209	-0.042
$\pi^*$	0.069	-0.026	-0.010	0.595	1.000	0.030	0.084	-0.050
r obs	0.553	0.521	-0.823	-0.548	0.030	1.000	-0.003	-0.022
y obs	0.343	-0.206	0.261	0.209	0.084	-0.003	1.000	-0.141
$y^*$	-0.062	0.009	-0.016	-0.042	-0.050	-0.022	-0.141	1.000

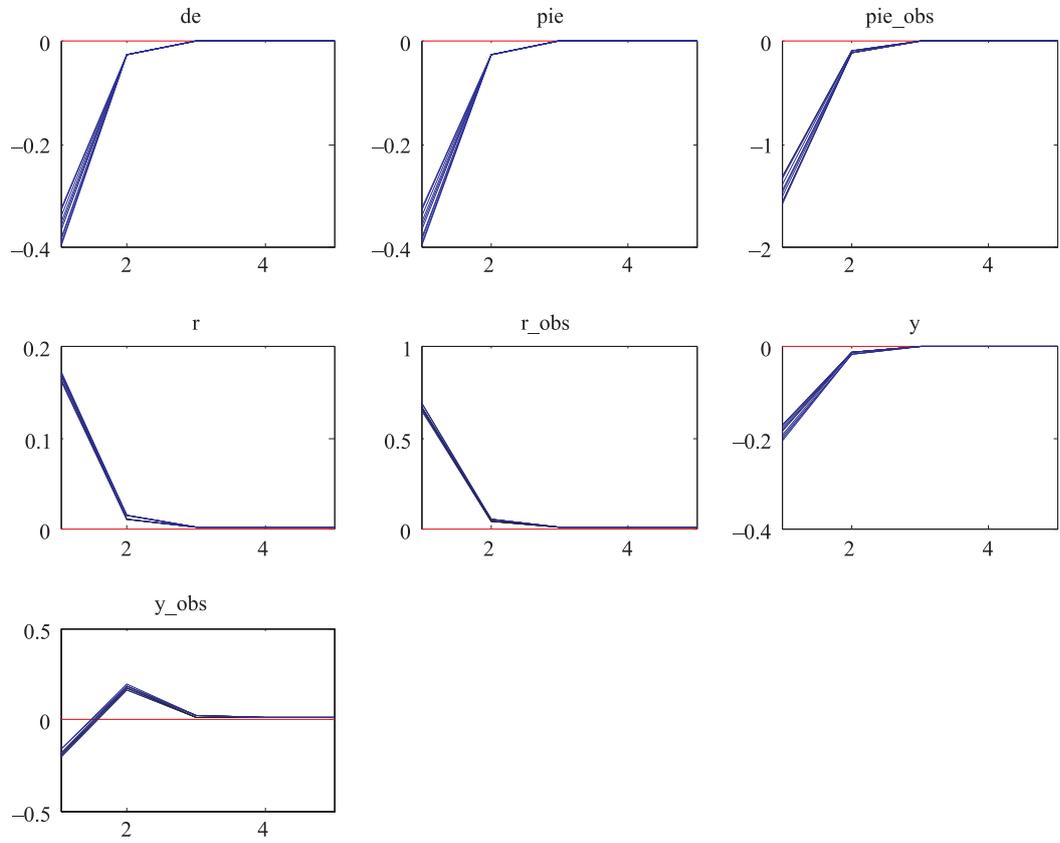
**Autocorrelation of simulated variables**

Variable	1	2	3	4	5
z	0.603	0.348	0.206	0.107	0.050
$\Delta e$	-0.245	-0.054	-0.046	-0.009	-0.041
$\Delta s$	0.145	0.062	0.024	0.019	0.005
$\pi$ obs	0.449	0.134	0.066	0.066	0.045
$\pi^*$	0.457	0.204	0.058	0.034	0.018
r obs	0.426	0.202	0.087	0.035	0.007
y obs	-0.010	0.026	-0.019	-0.020	-0.001
$y^*$	0.962	0.923	0.884	0.848	0.815

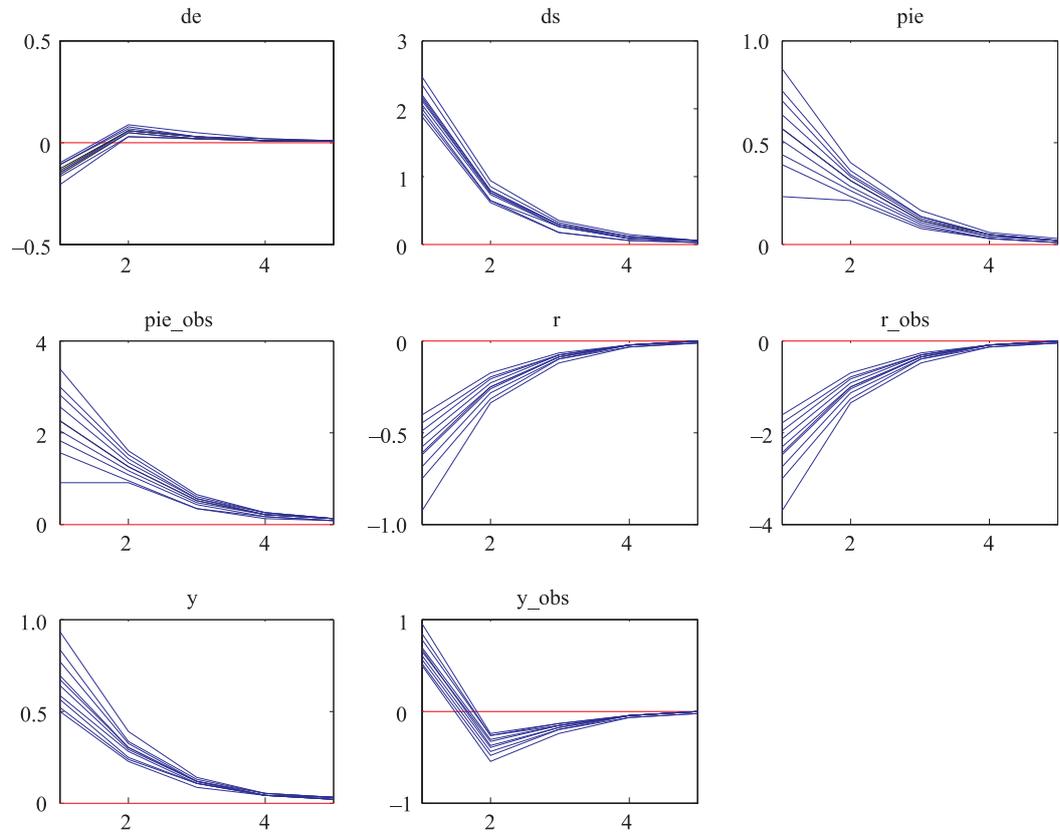
*Appendix 4*

**Bayesian estimation of impulse responses to shocks (1 000 000 MH simulations)**

**Response to monetary shock  $\varepsilon^r$**

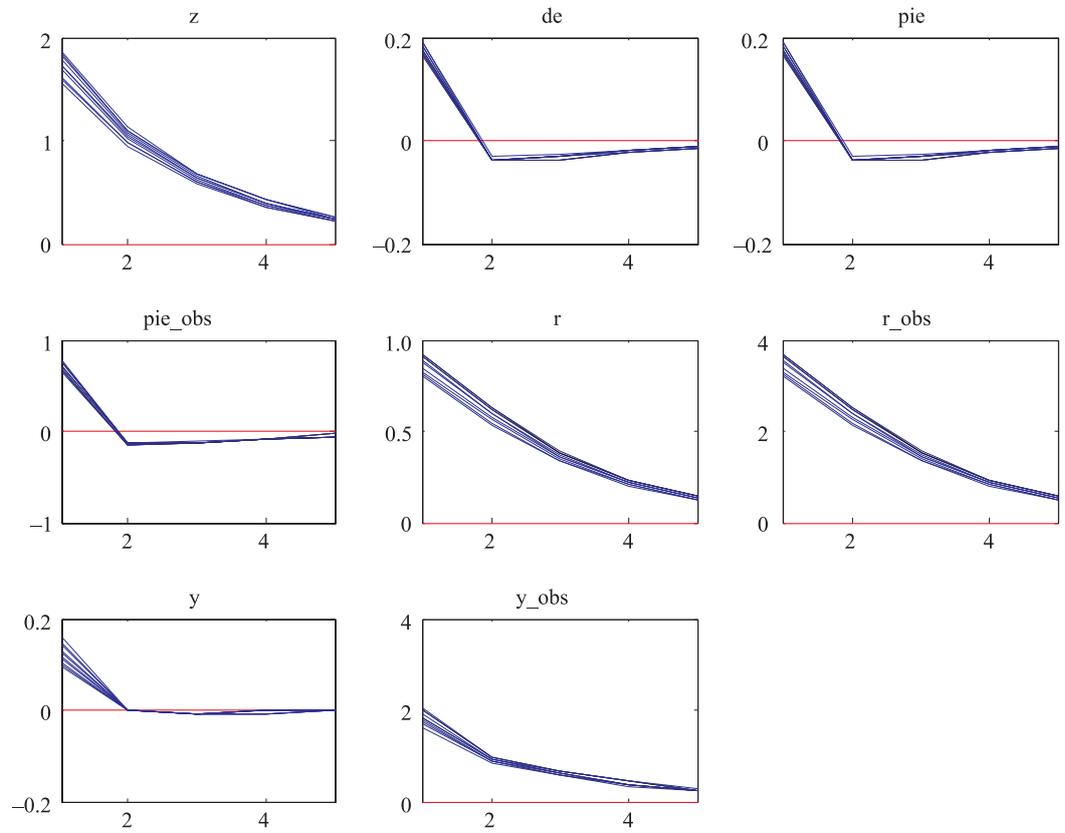


**Response to terms of trade shock  $\varepsilon^s$**

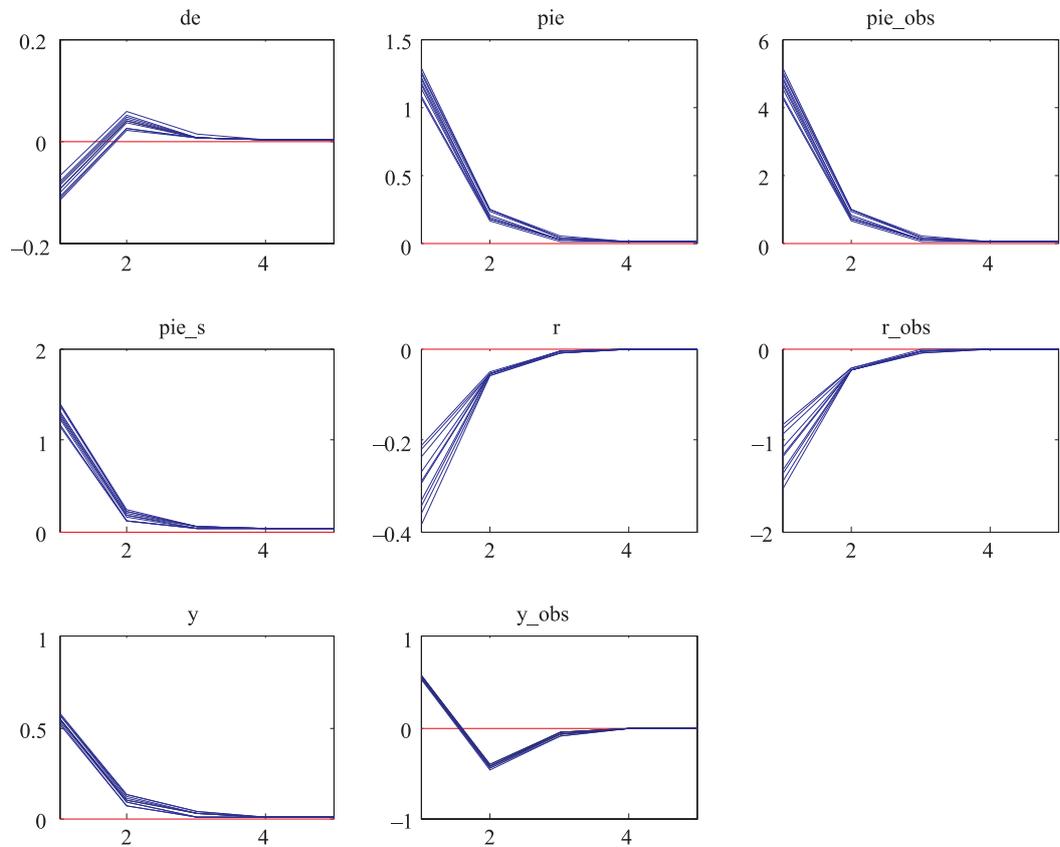


Notes:  $y$ ,  $r$ , and  $pie$  are output, interest rate and CPI inflation deviations from the steady state ( $\tilde{y}_t$ ,  $r_t$ , and  $\pi_t$  respectively, in the model equations). The steady state is the mean value over the estimation period.  $de$  stands for the exchange rate percentage deviation from the parity level to SDR and EUR in the respective periods ( $\Delta e_t$ ).  $ds$  is the terms of trade change ( $\Delta s_t$ ).  $r$ ,  $pie$ ,  $de$ , and  $ds$  are at quarterly frequencies.  $pie\_obs$  and  $r\_obs$  are the respective  $pie$  and  $r$  in annual terms.  $y\_obs = y - y(-1) + z$ .

**Response to productivity shock  $\varepsilon^z$**



**Response to foreign inflation shock  $\varepsilon \pi^*$**



Notes:  $y$ ,  $r$ ,  $pie$ , and  $pie\_s$  are output, interest rate, CPI inflation, and foreign inflation deviations from the steady state ( $\tilde{y}_t$ ,  $r_t$ ,  $\pi_t$ , and  $\pi_t^*$  respectively, in the model equations). The steady state is the mean value over the estimation period.  $de$  stands for the exchange rate percentage deviation from the parity level to SDR and EUR in the respective periods ( $\Delta e_t$ ).  $ds$  is the terms of trade change ( $\Delta s_t$ ).  $z$  is the technology growth rate ( $z_t$ ).  $r$ ,  $pie$ ,  $de$ , and  $ds$  are at quarterly frequencies.  $pie\_obs$  and  $r\_obs$  are the respective  $pie$  and  $r$  in annual terms.  $y\_obs = y - y(-1) + z$ .

## Appendix 5

**Derivation of the small open DSGE model**
**Households**

A representative household of a small open economy maximises its utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t / A_t, N_t) \quad [1]$$

where  $N_t$  denotes hours worked,  $A_t$  is the world technology process, and  $C_t$  is a composite consumption index defined as

$$C_t \equiv \left[ (1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad [2].$$

$C_{H,t}$  in its turn, is an index of consumption of domestic goods represented by the CES function

$$C_{H,t} \equiv \left( \int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where  $j \in [0, 1]$  denotes a differentiated good on the unit interval.  $C_{F,t}$  is an index of imported goods defined by

$$C_{F,t} \equiv \left( \int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

where  $C_{i,t}$  stands for an index of goods imported from country  $i$  and consumed by domestic households. As in the case of consumption of domestic goods, the index of imports is given by the CES function

$$C_{i,t} \equiv \left( \int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Parameter  $\varepsilon > 1$  implies the elasticity of substitution between goods produced within a specific country.  $\alpha \in [0, 1]$  measures a degree of openness which is commonly defined as share of imports in GDP. Parameter  $\eta > 0$  denotes the substitutability between domestic and foreign goods from the standpoint of the domestic consumer, while  $\gamma$  denotes the substitutability between goods imported from different markets.

The household maximises its utility defined in [1] subject to a borrowing constraint

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + D_t \leq D_{t-1} R_t + W_t N_t + T_t \quad [3]$$

for  $t = 0, 1, 2, \dots$  where  $P_{H,t}(j)$  is the price of differentiated domestic good  $j$  and  $P_{i,t}(j)$  is the price of differentiated good  $j$  imported from country  $i$ .  $R_t$  is return on

investment  $D_{t-1}$  held at the end of period  $t - 1$  (including shares in firms). Finally,  $W_t$  stands for the nominal wage, and  $T_t$  denotes lump-sum transfers (taxes).

Solving for optimal allocation between domestic and imported goods renders the respective demand functions

$$C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \quad [4]$$

$$C_{i,t}(j) = \left( \frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\varepsilon} C_{i,t}$$

for  $i, j \in [0, 1]$  where  $P_{H,t} \equiv \left( \int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$  refers to the price index of domestically produced goods and  $P_{i,t} \equiv \left( \int_0^1 P_{i,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$  is a price index (in domestic currency terms) for goods purchased from country  $i \in [0, 1]$ . Substituting definitions of price and quantity indexes  $P_{H,t}$ ,  $C_{H,t}$ ,  $P_{i,t}$ , and  $C_{i,t}$  into optimal allocations in [4] gives  $\int_0^1 P_{H,t}(j)C_{H,t}(j)dj = P_{H,t}C_{H,t}$  and  $\int_0^1 P_{i,t}(j)C_{i,t}(j)dj = P_{i,t}C_{i,t}$ .

Optimal choice of imports from country  $i$  yields

$$C_{i,t} = \left( \frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \quad [5]$$

for all  $i \in [0, 1]$  where  $P_{F,t} \equiv \left( \int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$  denotes the price index for imported goods, denominated in domestic currency. Combining the optimality condition in [5] with the definitions of  $P_{F,t}$ , and  $C_{F,t}$  yields the total expenditures on imported goods expressed as  $\int_0^1 P_{i,t}C_{i,t}di = P_{F,t}C_{F,t}$ .

To derive the optimal consumption allocation between domestic and imported goods, we consider profit maximisation problem of firms that buy quantities  $C_{H,t}$  and  $C_{F,t}$  of domestic and foreign produced goods and combine them into a composite good that is used for consumption by the households. These firms maximise profits in a perfectly competitive environment:

$$\max_{C_t, C_{H,t}, C_{F,t}} P_t C_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t}$$

$$\text{subject to } C_t = \left[ (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}.$$

Plugging the constraint into the profit equation and denoting it by  $f$  yields

$$f = P_t \left[ (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} - P_{H,t} C_{H,t} - P_{F,t} C_{F,t}.$$

Derivation of the first-order condition with respect to  $C_{H,t}$  results in

$$\frac{\partial f}{\partial C_{H,t}} = P_t \frac{\eta}{\eta-1} \left[ (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}-1} (1-\alpha)^{\frac{1}{\eta}} \frac{\eta-1}{\eta} C_{H,t}^{-\frac{1}{\eta}} - P_{H,t} = 0.$$

Simplifying and using the definition for  $C_t$  gives

$$P_t C_t^{\frac{1}{\eta}} (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{-\frac{1}{\eta}} = P_{H,t}$$

or rearranging

$$C_{H,t} = (1-\alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t.$$

In the same way, derivation of the first-order condition with respect to  $C_{F,t}$  implies

$$C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t.$$

Plugging the derived expressions for  $C_{H,t}$  and  $C_{F,t}$  into the definition for  $C_t$  yields

$$C_t = \left[ (1-\alpha)^{\frac{1}{\eta}} \left( (1-\alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left( \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

or rearranging

$$C_t^{\frac{\eta-1}{\eta}} = (1-\alpha)^{\frac{1}{\eta}} \left( (1-\alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left( \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \right)^{\frac{\eta-1}{\eta}},$$

opening the brackets

$$C_t^{\frac{\eta-1}{\eta}} = (1-\alpha)^{\frac{1}{\eta}} (1-\alpha)^{\frac{\eta-1}{\eta}} \left( \frac{P_{H,t}}{P_t} \right)^{1-\eta} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \alpha^{\frac{\eta-1}{\eta}} \left( \frac{P_{F,t}}{P_t} \right)^{1-\eta} C_t^{\frac{\eta-1}{\eta}},$$

cancelling  $C_t$  and simplifying

$$1 = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{1-\eta} + \alpha \left( \frac{P_{F,t}}{P_t} \right)^{1-\eta}$$

which yields the equation for the CPI

$$P_t \equiv \left[ (1 - \alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

For further convenience, let us rewrite the derived equations

$$C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t; \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t;$$

$$P_t \equiv \left[ (1 - \alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad [6].$$

Thus, total domestic household expenditures on consumption are  $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$ . Given this, one period budget constraint [3] can be reformulated as

$$P_tC_t + D_t \leq D_{t-1}R_t + W_tN_t + T_t \quad [7].$$

To obtain the intratemporal optimality condition, we express the household's utility maximisation problem as a Lagrangian,  $L$ :

$$L_0 = E_0 \sum_{t=0}^{\infty} \left( \beta^t \left[ \frac{(C_t / A_t)^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] - \lambda_t (P_tC_t + D_t - D_{t-1}R_t - W_tN_t - T_t) \right)$$

where  $\sigma$  and  $\varphi$  represent household's risk aversion and labour supply aversion respectively.

The relevant first-order conditions are:

$$\frac{\partial L_t}{\partial D_t} = -\lambda_t + E_t \lambda_{t+1} R_{t+1} = 0 \Rightarrow \lambda_t = E_t \lambda_{t+1} R_{t+1}$$

$$\frac{\partial L_t}{\partial C_t} = \beta^t \frac{1}{A_t^{1-\sigma}} C_t^{-\sigma} - \lambda_t P_t = 0 \Rightarrow \lambda_t = \beta^t \frac{1}{A_t^{1-\sigma} P_t} C_t^{-\sigma}.$$

Eliminating  $\lambda_t$  yields

$$\beta^t \frac{1}{A_t^{1-\sigma} P_t} C_t^{-\sigma} = E_t \left( \beta^{t+1} \frac{1}{A_{t+1}^{1-\sigma} P_{t+1}} C_{t+1}^{-\sigma} R_{t+1} \right)$$

or

$$\frac{C_t^{-\sigma}}{A_t^{-\sigma}} = \beta E_t \left( \frac{C_{t+1}^{-\sigma} P_t}{A_{t+1}^{-\sigma} P_{t+1}} \frac{A_t}{A_{t+1}} R_{t+1} \right).$$

Denoting  $\tilde{C}_t \equiv \frac{C_t}{A_t}$ , the latter can be expressed as

$$\tilde{C}_t^{-\sigma} = \beta E_t \left( \tilde{C}_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \frac{A_t}{A_{t+1}} R_{t+1} \right) \quad [8]$$

which is the Euler equation.

Rearranging the latter yields the one-period stochastic discount factor

$$\beta \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{A_t}{A_{t+1}} \equiv Q_{t,t+1} \quad [9]$$

Using  $X_t^\alpha \cong X^\alpha (1 + \alpha x_t)$ , log-linearisation of [8] yields

$$\tilde{C}^{-\sigma} (1 - \sigma \tilde{c}_t) = \beta \tilde{C}^{-\sigma} R (1 - \sigma E_t \tilde{c}_{t+1} + p_t - E_t p_{t+1} + a_t - E_t a_{t+1} + r_t)$$

and making use of the steady state condition  $\beta^{-1} = R$  which follows from [8], we obtain

$$-\sigma \tilde{c}_t = -\sigma E_t \tilde{c}_{t+1} + p_t - E_t p_{t+1} + a_t - E_t a_{t+1} + r_t$$

or

$$\sigma \tilde{c}_t = \sigma E_t \tilde{c}_{t+1} - p_t + E_t p_{t+1} - a_t + E_t a_{t+1} - r_t \Rightarrow \tilde{c}_t = E_t \tilde{c}_{t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}) + \frac{1}{\sigma} E_t \{\Delta a_{t+1}\} \quad [10]$$

where lowercase letters stand for deviations from the steady state of the respective variables and  $\pi_{t+1} \equiv p_{t+1} - p_t$  is CPI inflation.

Optimal labour supply choice implies

$$\frac{\partial L_t}{\partial N_t} = -\beta' N_t^\varphi + \lambda_t W_t = 0$$

or substituting for  $\lambda_t$

$$\frac{W_t C_t^{-\sigma}}{A_t^{1-\sigma} P_t} = N_t^\varphi \Rightarrow \frac{W_t}{P_t A_t} = \left( \frac{C_t}{A_t} \right)^\sigma N_t^\varphi \Rightarrow \frac{W_t}{P_t A_t} = \tilde{C}_t^\sigma N_t^\varphi \quad [11].$$

Denoting  $\tilde{W}_t \equiv \frac{W_t}{P_t A_t}$  yields

$$\tilde{W}_t = \tilde{C}_t^\sigma N_t^\varphi$$

which in log-linearised form implies

$$\tilde{w}_t = \sigma \tilde{c}_t + \varphi n_t \quad [12].$$

### Identities between inflation, exchange rates and terms of trade

Next, several identities linking inflation, exchange rates and terms of trade are defined. Bilateral terms of trade between the domestic economy and country  $i$  is given by

$$S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$$

which is nothing but the price of country  $i$ 's goods in terms of home goods. Consequently, the effective terms of trade are defined as

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left( \int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}.$$

Using  $S_t = e^{\log S_t} \approx 1 + \log S_t = 1 + s_t$ , where  $s_t \equiv \log S_t = p_{F,t} - p_{H,t}$ , the last expression can be transformed as

$$\begin{aligned} S_t &= \left( \int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left( \int_0^1 e^{\log(S_{i,t}^{1-\gamma})} di \right)^{\frac{1}{1-\gamma}} \approx \left( \int_0^1 [1 + (1-\gamma) \log S_{i,t}] di \right)^{\frac{1}{1-\gamma}} = \left( 1 + \int_0^1 (1-\gamma) \log S_{i,t} di \right)^{\frac{1}{1-\gamma}} \approx \\ &\approx 1 + \frac{1}{1-\gamma} \int_0^1 (1-\gamma) \log S_{i,t} di = 1 + \int_0^1 \log S_{i,t} di = 1 + \int_0^1 s_{i,t} di = 1 + s_t. \end{aligned} \quad ^2$$

Thus the effective terms of trade can be approximated (up to first order) around a symmetric steady state satisfying  $S_{i,t} = 1$  for all  $i \in [0, 1]$  by

$$s_t = \int_0^1 s_{i,t} di \quad [13].$$

Analogously, log-linearisation of the expression for the CPI around the symmetric steady state gives

$$\begin{aligned} P_t &= e^{\log P_t} \approx \\ &\approx 1 + \log P_t = 1 + p_t = \left[ (1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} = \left[ (1-\alpha)(e^{\log P_{H,t}})^{1-\eta} + \alpha(e^{\log P_{F,t}})^{1-\eta} \right]^{\frac{1}{1-\eta}} \approx \\ &\approx \left[ (1-\alpha)(1 + \log P_{H,t})^{1-\eta} + \alpha(1 + \log P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \approx \left[ (1-\alpha)(1 + (1-\eta)p_{H,t}) + \alpha(1 + (1-\eta)p_{F,t}) \right]^{\frac{1}{1-\eta}} = \end{aligned}$$

<sup>2</sup> We use approximation  $(1 + x_t)^\alpha \approx 1 + \alpha x_t$ , where  $x_t$  is a real number close to zero.

$$\begin{aligned}
 &= \left[ 1 + (1-\alpha)(1-\eta)p_{H,t} + \alpha(1-\eta)p_{F,t} \right]^{\frac{1}{1-\eta}} \approx 1 + \frac{1}{1-\eta} \left( (1-\alpha)(1-\eta)p_{H,t} + \alpha(1-\eta)p_{F,t} \right) = \\
 &= 1 + (1-\alpha)p_{H,t} + \alpha p_{F,t}
 \end{aligned}$$

yielding

$$p_t \equiv (1-\alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t \quad [14].$$

From the last formula it follows that domestic inflation, i.e., the rate of change in the domestic goods price index,  $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ , and CPI inflation are related in the form

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \quad [15]$$

which implies that the inflation difference is proportional to the percent change in the terms of trade where the coefficient of proportionality is captured by the degree of openness  $\alpha$ .

Furthermore, it is assumed that the law of one price holds at a product level both for import and export prices, implying  $P_{i,t}(j) = \varepsilon_{i,t} P_{i,t}^i(j)$  for all  $i, j \in [0, 1]$ .  $\varepsilon_{i,t}$  is the bilateral nominal exchange rate, i.e., the price of country  $i$ 's currency in terms of the domestic currency, whereas  $P_{i,t}^i(j)$  is the price of country  $i$ 's good  $j$  denominated in its own currency terms. Applying the law-of-one-price assumption to the definition of  $P_{i,t}$  results in  $P_{i,t} = \varepsilon_{i,t} P_{i,t}^i$  where  $P_{i,t}^i \equiv \left( \int_0^1 P_{i,t}^i(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$  stands for country  $i$ 's domestic price index.

Substituting the same assumption into the expression for  $P_{F,t}$  and log-linearising around the symmetric steady state gives

$$\begin{aligned}
 P_{F,t} &= e^{\log P_{F,t}} \approx 1 + \log P_{F,t} = 1 + p_{F,t} = \left( \int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left( \int_0^1 (\varepsilon_{i,t} P_{i,t}^i)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \\
 &= \left( \int_0^1 \left( \varepsilon_{i,t} \left( \int_0^1 P_{i,t}^i(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left( \int_0^1 \left( \varepsilon_{i,t} \left( \int_0^1 e^{(1-\varepsilon) \log P_{i,t}^i(j)} dj \right)^{\frac{1}{1-\varepsilon}} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \\
 &\approx \left( \int_0^1 \left( \varepsilon_{i,t} \left( \int_0^1 [1 + (1-\varepsilon) \log P_{i,t}^i(j)] dj \right)^{\frac{1}{1-\varepsilon}} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left( \int_0^1 \left( \varepsilon_{i,t} \left( 1 + \int_0^1 [(1-\varepsilon) \log P_{i,t}^i(j)] dj \right)^{\frac{1}{1-\varepsilon}} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \\
 &\approx \left( \int_0^1 \left( \varepsilon_{i,t} \left( 1 + \frac{1}{1-\varepsilon} \int_0^1 [(1-\varepsilon) \log P_{i,t}^i(j)] dj \right) \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left( \int_0^1 \left( \varepsilon_{i,t} \left( 1 + \int_0^1 [P_{i,t}^i(j)] dj \right) \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \\
 &= \left( \int_0^1 (\varepsilon_{i,t} (1 + p_{i,t}^i))^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left( \int_0^1 (e^{\log \varepsilon_{i,t}} (1 + p_{i,t}^i))^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \approx \left( \int_0^1 ((1 + \log(\varepsilon_{i,t})) (1 + p_{i,t}^i))^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}
 \end{aligned}$$

$$\begin{aligned} &\approx \left( \int_0^1 \left( (1 + e_{i,t})(1 + p_{i,t}^i) \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \approx \left( \int_0^1 (1 + e_{i,t} + p_{i,t}^i)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \approx \left( \int_0^1 (1 + (1-\gamma)(e_{i,t} + p_{i,t}^i)) di \right)^{\frac{1}{1-\gamma}} \\ &= \left( 1 + (1-\gamma) \int_0^1 (e_{i,t} + p_{i,t}^i) di \right)^{\frac{1}{1-\gamma}} \approx \left( 1 + \int_0^1 (e_{i,t} + p_{i,t}^i) di \right) = 1 + e_t + p_t^* \end{aligned}$$

or

$$p_{F,t} = \int_0^1 (e_{i,t} + p_{i,t}^i) di = e_t + p_t^*$$

where  $p_{i,t}^i \equiv \int_0^1 p_{i,t}^i(j) dj$  denotes country  $i$ 's log domestic price index in terms of its own currency,  $e_t \equiv \int_0^1 e_{i,t} di$  is the log effective nominal exchange rate, and  $p_t^* \equiv \int_0^1 p_{i,t}^i di$  stands for the log world price index.

Plugging the terms of trade definition into the last relationship yields

$$s_t = e_t + p_t^* - p_{H,t} \quad [16].$$

The next step is to derive a relationship linking the terms of trade and the real exchange rate. The bilateral real exchange rate with country  $i$  is defined as

$\Omega_{i,t} \equiv \frac{\varepsilon_{i,t} P_t^i}{P_t}$ , i.e., the ratio of the two countries' CPIs (both denominated in

domestic currency). Defining  $q_t \equiv \int_0^1 q_{i,t} di$  as the log effective real exchange rate, where  $q_{i,t} \equiv \log \Omega_{i,t}$ , results in

$$q_t = \int_0^1 (e_{i,t} + p_t^i - p_t) di = e_t + p_t^* - p_t = s_t + p_{H,t} - p_t.$$

Rewriting the CPI expression in the form

$$P_t^{1-\eta} = (1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}$$

and dividing both sides by  $P_{H,t}^{1-\eta}$  yields

$$\left( \frac{P_t}{P_{H,t}} \right)^{1-\eta} = (1-\alpha) + \alpha \left( \frac{P_{F,t}}{P_{H,t}} \right)^{1-\eta} = (1-\alpha) + \alpha S_t^{1-\eta}$$

or

$$\frac{P_t}{P_{H,t}} = [(1-\alpha) + \alpha S_t^{1-\eta}]^{\frac{1}{1-\eta}}.$$

Log-linearising the latter around a symmetric steady state

$$\begin{aligned} \log\left(\frac{P_t}{P_{H,t}}\right) &= \log P_t - \log P_{H,t} = p_t - p_{H,t} = \frac{1}{1-\eta} \log[(1-\alpha) + \alpha S_t^{1-\eta}] = \frac{1}{1-\eta} \log[1 + \alpha(S_t^{1-\eta} - 1)] \approx \\ &\approx \frac{1}{1-\eta} \alpha(S_t^{1-\eta} - 1) = \frac{1}{1-\eta} \alpha(e^{(1-\eta)\log S_t} - 1) \approx \frac{1}{1-\eta} \alpha(1 + (1-\eta)\log S_t - 1) = \alpha s_t, \end{aligned}$$

which results in  $p_t - p_{H,t} = \alpha s_t$ , and plugging in the last expression for  $q_t$  yields

$$q_t = e_t + p_t^* - p_t = s_t + p_{H,t} - p_t = (1-\alpha)s_t \quad [17].$$

Furthermore, a few identities related to international risk sharing are derived. Condition [9] which characterises the domestic economy must also hold for any country  $i$ , accounting for the exchange rate

$$\beta \left( \frac{\tilde{C}_{t+1}^i}{\tilde{C}_t^i} \right)^{-\sigma} \frac{P_t^i}{P_{t+1}^i} \frac{A_t}{A_{t+1}} \frac{\varepsilon_t^i}{\varepsilon_{t+1}^i} = Q_{t,t+1} \quad [18].$$

Equating left hand sides of [9] and [18] and using definition of the real exchange rate yields

$$\begin{aligned} \beta \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \left( \frac{A_t}{A_{t+1}} \right) \left( \frac{P_t}{P_{t+1}} \right) &= \beta \left( \frac{\tilde{C}_{t+1}^i}{\tilde{C}_t^i} \right)^{-\sigma} \left( \frac{A_t}{A_{t+1}} \right) \left( \frac{P_t^i}{P_{t+1}^i} \right) \left( \frac{\varepsilon_t^i}{\varepsilon_{t+1}^i} \right), \\ \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \left( \frac{P_{t+1}^i}{P_{t+1}} \right) \varepsilon_{t+1}^i &= \left( \frac{\tilde{C}_t}{\tilde{C}_t^i} \right)^{-\sigma} \left( \frac{P_t^i}{P_t} \right) \varepsilon_t^i = \dots = \left( \frac{\tilde{C}_0}{\tilde{C}_0^i} \right)^{-\sigma} \left( \frac{P_0^i}{P_0} \right) \varepsilon_0^i = const_i \end{aligned}$$

which is referred to as the "initial consumption ratio".

Rearranging yields

$$\tilde{C}_t = \frac{1}{const_i^{\frac{1}{\sigma}}} \tilde{C}_t^i \Omega_{i,t}^{\frac{1}{\sigma}} = \mathcal{G}_i \tilde{C}_t^i \Omega_{i,t}^{\frac{1}{\sigma}} \quad [19]$$

where  $\mathcal{G}_i$  is a constant which depends on initial conditions regarding relative net asset positions. Assuming symmetric initial conditions, i.e., zero net foreign asset holdings and an ex-ante identical environment,  $\mathcal{G}_i = \mathcal{G} = 1$  for all  $i$ .

Expressing [19] in logs

$$\log \tilde{C}_t = \tilde{c}_t = \log \left( \tilde{C}_t^i \Omega_{i,t}^{\frac{1}{\sigma}} \right) = \log \tilde{C}_t^i + \frac{1}{\sigma} \log \Omega_{i,t} = \tilde{c}_t^i + \frac{1}{\sigma} q_{i,t}$$

and integrating over  $i$  results in

$$\int_0^1 \tilde{c}_t di = \tilde{c}_t = \int_0^1 \tilde{c}_t^i di + \int_0^1 \frac{1}{\sigma} q_{i,t} di = \tilde{c}_t^* + \frac{1}{\sigma} q_t = \tilde{c}_t^* + \left( \frac{1-\alpha}{\sigma} \right) s_t \quad [20]$$

where  $\tilde{c}_t^* \equiv \int_0^1 \tilde{c}_t^i di$  stands for stationary log world consumption, and the second equality is obtained by substituting the expression for  $q_t$  from [17]. The derived equation relates domestic consumption with world consumption and terms of trade.

### Firms

The domestic economy is populated by a continuum of firms  $j \in [0, 1]$ , where each one produces a differentiated good using the same technology, represented by the production function

$$Y_t(j) = A_t N_t(j)$$

where  $A_t$  is the level of technology and  $a_t \equiv \log A_t$  is described by the AR(1) process  $a_t = \rho_a a_{t-1} + v_t$ .

All firms face identical demand curves and take the aggregate price level and aggregate consumption index exogenously. Following the price setting mechanism proposed by Calvo (1983), each firm may change its price with probability  $1 - \theta$  every period, irrespective of the last time of adjustment. Thus, each period a fraction  $1 - \theta$  of firms reset their prices, whereas the rest  $\theta$  keep their prices unchanged. In this way,  $\theta$  represents price stickiness.

Next, the aggregate price level dynamics is described. Given that all firms resetting prices will choose the same price  $\bar{P}_{H,t}$ , the aggregate price level takes the form

$$P_{H,t} = \left[ \theta (P_{H,t-1})^{1-\varepsilon} + (1-\theta) (\bar{P}_{H,t})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

Dividing both sides by  $P_{H,t-1}$  yields

$$\Pi_{H,t}^{1-\varepsilon} = \theta + (1-\theta) \left( \frac{\bar{P}_{H,t}}{P_{H,t-1}} \right)^{1-\varepsilon} \quad [21]$$

where  $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$  is the gross inflation rate between  $t - 1$  and  $t$  of the domestically produced goods. Given that in a steady state with zero inflation  $\bar{P}_{H,t} = P_{H,t-1} = P_{H,t}$  for all  $t$ , log-linearisation of the last expression around the steady state results in

$$\pi_{H,t} = (1-\theta) (\bar{p}_{H,t} - p_{H,t-1}) \quad [22].$$

Equation [22] implies that inflation results from firms re-optimising their price each period so that it differs from the average  $t - 1$  period price in the economy. Therefore, to follow the inflation dynamics in the course of time, the next step is to clarify the factors underlying firms' price setting decisions.

A firm re-optimising in period  $t$  will choose the price  $\bar{P}_{H,t}$  to maximise the present market value of its profits generated while the price remains effective

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( \bar{P}_{H,t} Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\} = 0$$

subject to the set of demand constraints

$$Y_{t+k|t} = \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left( C_{H,t+k} + \int_0^1 C_{H,t+k}^i di \right) = \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \hat{C}_{H,t+k} \equiv Y_{t+k}^d(\bar{P}_{H,t}) \quad [23]$$

for  $k = 0, 1, 2, \dots$  where  $Q_{t,t+k} \equiv \beta^k (\tilde{C}_{t+k} / \tilde{C}_t)^{-\sigma} (A_t / A_{t+k})(P_t / P_{t+k})$  is the stochastic discount factor for nominal payoffs described above,  $\Psi_t(\cdot)$  is the cost function, and  $Y_{t+k|t}$  denotes the  $t+k$  period output of a firm that last reset its price in period  $t$ .

Plugging the expression for  $Y_{t+k|t}$  from [23] into the profit maximisation equation and denoting it by  $L$  yields

$$\begin{aligned} L &= \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( \bar{P}_{H,t} \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \hat{C}_{H,t+k} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\} = \\ &= \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( (\bar{P}_{H,t})^{1-\varepsilon} \left( \frac{1}{P_{H,t+k}} \right)^{-\varepsilon} \hat{C}_{H,t+k} - \Psi_{t+k} \left( \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \hat{C}_{H,t+k} \right) \right) \right\}. \end{aligned}$$

The first order condition with respect to  $\bar{P}_{H,t}$  implies

$$\frac{\partial L}{\partial \bar{P}_{H,t}} = \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( (1-\varepsilon) \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \hat{C}_{H,t+k} - \Psi_{t+k|t} \cdot (-\varepsilon) \cdot \frac{(\bar{P}_{H,t})^{-\varepsilon-1}}{(P_{H,t+k})^{-\varepsilon}} \hat{C}_{H,t+k} \right) \right\} = 0$$

or using [23]

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( (1-\varepsilon) Y_{t+k|t} + \varepsilon \Psi_{t+k|t} \frac{Y_{t+k|t}}{\bar{P}_{H,t}} \right) \right\} = 0.$$

Multiplying both terms by  $-\bar{P}_{H,t} / (\varepsilon - 1)$  gives

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} (\bar{P}_{H,t} - \Lambda \Psi_{t+k|t}) \right\} = 0 \quad [24]$$

where  $\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})$  stands for the nominal marginal cost in period  $t+k$  for a firm that last reset its price in period  $t$  and  $\Lambda \equiv \frac{\varepsilon}{\varepsilon-1}$ .

In case of no price rigidities, i.e., at  $\theta = 0$ , the last condition reduces to the standard optimal price-setting condition under flexible prices

$$\bar{P}_{H,t} = \Lambda \psi_{t|t}$$

where  $\Lambda$  may thus be regarded as the desired or frictionless mark-up.

Before linearising the optimal price-setting condition [24], it is useful to rewrite it in terms of variables that have a well-defined value in the steady state. In particular, dividing by  $P_{H,t-1}$  and denoting  $\Pi_{H,t,t+k} \equiv P_{H,t+k} / P_{H,t}$ , equation [24] takes the form

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left( \frac{\bar{P}_{H,t}}{P_{H,t-1}} - \frac{\Lambda \psi_{t+k|t}}{P_{H,t-1}} \frac{P_{H,t+k}}{P_{H,t+k}} \right) \right\} = 0$$

or

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left( \frac{\bar{P}_{H,t}}{P_{H,t-1}} - \Lambda MC_{t+k|t} \Pi_{H,t-1,t+k} \right) \right\} = 0 \quad [25]$$

where  $MC_{t+k|t} \equiv \psi_{t+k|t} / P_{H,t+k}$  is the real marginal cost in period  $t+k$  for a firm whose price was last set in period  $t$ .

Since prices are constant in a steady state implying  $\bar{P}_{H,t} = P_{H,t+k}$ , it follows that  $Y_{t+k|t} = Y$  and  $MC_{t+k|t} = MC$  due to the fact that all firms will produce an equal quantity of output. Moreover, from  $Q_{t,t+k} \equiv \beta^k (\tilde{C}_{t+k} / \tilde{C}_t)^{-\sigma} (A_t / A_{t+k})(P_t / P_{t+k})$  it follows that the condition  $Q_{t,t+k} = \beta^k$  must hold in the steady state, meaning  $MC = 1/\Lambda$ . A first-order Taylor expansion of [25] around the zero inflation steady state yields

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k (1 + q_{t,t+k}) Y (1 + \hat{y}_{t+k|t}) \left( e^{\frac{\log \bar{P}_{H,t}}{P_{H,t-1}}} - e^{\log \Lambda MC_{t+k|t} P_{H,t+k} / P_{H,t}} \right) \right\} = 0;$$

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k (1 + q_{t,t+k}) (1 + \hat{y}_{t+k|t}) \left( e^{\log \bar{P}_{H,t} - \log P_{H,t-1}} - e^{\log \Lambda + \log MC_{t+k|t} + \log P_{H,t+k} - \log P_{H,t}} \right) \right\} = 0;$$

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k (1 + q_{t,t+k}) (1 + \hat{y}_{t+k|t}) \left( e^{\bar{P}_{H,t} - P_{H,t-1}} - e^{\mu + mc_{t+k|t} + P_{H,t+k} - P_{H,t}} \right) \right\} = 0;$$

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k (1 + q_{t,t+k}) (1 + \hat{y}_{t+k|t}) \left( 1 + \bar{P}_{H,t} - P_{H,t-1} - \underbrace{(1 - mc + mc_{t+k|t})}_{\hat{m}c_{t+k,k}} + P_{H,t+k} - P_{H,t} \right) \right\} = 0;$$

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k (1 + q_{t,t+k})(1 + \hat{y}_{t+k|t}) (\bar{p}_{H,t} - p_{H,t-1} - \hat{m}c_{t+k|t} - (p_{H,t+k} - p_{H,t-1})) \right\} = 0.$$

Since in the steady state

$$(\bar{p}_{H,t} - p_{H,t-1} - \hat{m}c_{t+k|t} - (p_{H,t+k} - p_{H,t-1})) = 0,$$

it follows that

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k (\bar{p}_{H,t} - p_{H,t-1} - \hat{m}c_{t+k|t} - (p_{H,t+k} - p_{H,t-1})) \right\} = 0;$$

or rearranging

$$(\bar{p}_{H,t} - p_{H,t-1}) \sum_{k=0}^{\infty} \theta^k \beta^k = \sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k (\hat{m}c_{t+k|t} + (p_{H,t+k} - p_{H,t-1})) \right\} = 0.$$

Using  $\sum_{k=0}^{\infty} \theta^k \beta^k = \frac{1}{1 - \theta\beta}$ , the latter yields

$$\frac{(\bar{p}_{H,t} - p_{H,t-1})}{1 - \theta\beta} = \sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k (\hat{m}c_{t+k|t} + (p_{H,t+k} - p_{H,t-1})) \right\} = 0$$

or

$$\bar{p}_{H,t} - p_{H,t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \hat{m}c_{t+k|t} + (p_{H,t+k} - p_{H,t-1}) \right\} \quad [26]$$

where  $\hat{m}c_{t+k|t} \equiv mc_{t+k|t} - mc$  stands for the log deviation of marginal cost from its steady state value  $mc = -\mu$  and  $\mu \equiv \log \Lambda$  is the log of the desired gross mark-up.

## Equilibrium

### *The demand side*

Goods market clearing in the domestic economy requires

$$\frac{Y_t(j)}{A_t} = \frac{C_{H,t}(j)}{A_t} + \int_0^1 \frac{C_{H,t}^i(j)}{A_t} di$$

for all  $j \in [0, 1]$  and all  $t$  where  $C_{H,t}^i(j)$  stands for country  $i$ 's demand for the domestically produced good  $j$ .

Let us denote  $\tilde{Y}_t(j) \equiv \frac{Y_t(j)}{A_t}$ ;  $\tilde{C}_{H,t}(j) \equiv \frac{C_{H,t}(j)}{A_t}$ ;  $\tilde{C}_{H,t}^i(j) \equiv \frac{C_{H,t}^i(j)}{A_t}$ .

Using [4] and [6], domestic consumption can be rewritten in the form

$$C_{H,t}(j) = \underbrace{\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}}_{(4)} C_{H,t} = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \underbrace{(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}}_{(6)} C_t$$

or

$$\tilde{C}_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \tilde{C}_{H,t} = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \tilde{C}_t.$$

Assuming symmetric preferences across countries and using [4], [5], and [6], country  $i$ 's demand for the domestically produced good  $j$  can be expressed as

$$\tilde{C}_{H,t}^i(j) = \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} \tilde{C}_t^i.$$

Plugging the last two expressions into the goods market clearing equation yields

$$\frac{Y_t(j)}{A_t} = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \times \left[ (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \frac{C_t}{A_t} + \alpha \int_0^1 \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} \frac{C_t^i}{A_t} di \right]$$

or

$$\tilde{Y}_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \times \left[ (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \tilde{C}_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} \tilde{C}_t^i di \right] \quad [27].$$

Substituting [27] into the aggregate domestic output equation given by

$$\tilde{Y}_t \equiv \left[ \int_0^1 \tilde{Y}_t(j)^{\frac{1-\varepsilon}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ gives}$$

$$\begin{aligned} \tilde{Y}_t &\equiv \left[ \int_0^1 \left\{ \underbrace{\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}}_{=1} \times \left[ (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \tilde{C}_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} \tilde{C}_t^i di \right] \right\}^{1-\frac{1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} = \\ &= \left\{ \left[ \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon \frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[ \frac{1}{(P_{H,t})^{-\varepsilon}} \int_0^1 (P_{H,t}(j))^{1-\varepsilon} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} = \left\{ P_{H,t} \equiv \left( \int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \right\} \right\} = \\ &= \left\{ \left[ \frac{1}{(P_{H,t})^{-\varepsilon}} \int_0^1 (P_{H,t}(j))^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}(-\varepsilon)} = 1 \right\} \end{aligned}$$

$$\begin{aligned}
 &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \tilde{C}_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} \tilde{C}_t^i di = \\
 &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[ (1-\alpha)\tilde{C}_t + \alpha \int_0^1 \left(\frac{\varepsilon_{i,t} P_{F,t}^i}{P_{H,t}}\right)^\gamma \left(\frac{P_{H,t}}{P_t}\right)^\eta \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} \tilde{C}_t^i di \right] = \\
 &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[ (1-\alpha)\tilde{C}_t + \alpha \int_0^1 \left(\frac{\varepsilon_{i,t} P_{F,t}^i}{P_{H,t}}\right)^\gamma \left(\frac{\varepsilon_{i,t} P_{F,t}^i}{P_{H,t}}\right)^{-\eta} (\varepsilon_{i,t})^\eta \left(\frac{1}{P_t}\right)^\eta \left(\frac{1}{P_t^i}\right)^{-\eta} \tilde{C}_t^i di \right] = \left\{ \Omega_{i,t} \equiv \frac{\varepsilon_{i,t} P_t^i}{P_t} \right\} = \\
 &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[ (1-\alpha)\tilde{C}_t + \alpha \int_0^1 \left(\frac{\varepsilon_{i,t} P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta} \Omega_{i,t}^\eta \tilde{C}_t^i di \right] = \left\{ (19) \Rightarrow \tilde{C}_t = \tilde{C}_t^i \Omega_{i,t}^{\frac{1}{\sigma}} \Rightarrow \tilde{C}_t^i = \tilde{C}_t \Omega_{i,t}^{-\frac{1}{\sigma}} \right\} = \\
 &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[ (1-\alpha)\tilde{C}_t + \alpha \int_0^1 \underbrace{\left(\frac{P_{i,t}}{P_{H,t}}\right)}_{S_{i,t}} \underbrace{\left(\frac{\varepsilon_{i,t} P_{F,t}^i}{P_{i,t}}\right)}_{S_{i,t}^i} \right]^{\gamma-\eta} \Omega_{i,t}^{\eta-\frac{1}{\sigma}} \tilde{C}_t di = \\
 &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \tilde{C}_t \left[ (1-\alpha) + \alpha \int_0^1 (S_{i,t}^i S_{i,t})^{\gamma-\eta} \Omega_{i,t}^{\eta-\frac{1}{\sigma}} di \right] \tag{28}
 \end{aligned}$$

where the last equality follows from [19], and where  $S_t^i$  denotes country  $i$ 's effective terms of trade, while  $S_{i,t}$  stands for the bilateral terms of trade between the domestic economy and country  $i$ .

Given  $\int_0^1 s_t^i di = 0$ , the first-order log-linear approximation to [28] around the steady state implies

$$\begin{aligned}
 \tilde{Y}_t &= e^{\log \tilde{Y}_t} \approx 1 + \log \tilde{Y}_t = 1 + \tilde{y}_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \tilde{C}_t \left[ (1-\alpha) + \alpha \int_0^1 (S_{i,t}^i S_{i,t})^{\gamma-\eta} \Omega_{i,t}^{\eta-\frac{1}{\sigma}} di \right] = \\
 &\approx (1-\eta)(\log P_{H,t} - \log P_t) (1 + \log \tilde{C}_t) \left[ (1-\alpha) + \alpha \int_0^1 \left[ 1 + (\gamma-\eta)(\log S_{i,t}^i + \log S_{i,t}) \right] \left[ 1 + \left(\eta - \frac{1}{\sigma}\right) \log \Omega_{i,t} \right] di \right] = \\
 &= \left( 1 - \eta \underbrace{\left(\frac{p_{H,t} - p_t}{p_t}\right)}_{=(14)=\alpha_t} \right) (1 + \tilde{c}_t) \left[ (1-\alpha) + \alpha \int_0^1 \left[ 1 + (\gamma-\eta)(s_{i,t}^i + s_{i,t}) \right] \left[ 1 + \left(\eta - \frac{1}{\sigma}\right) q_{i,t} \right] di \right] = \\
 &= (1 - \eta \alpha_t) (1 + \tilde{c}_t) \left[ (1-\alpha) + \alpha \left[ 1 + (\gamma-\eta) \int_0^1 s_{i,t} di + \left(\eta - \frac{1}{\sigma}\right) \int_0^1 q_{i,t} di \right] \right] =
 \end{aligned}$$

$$\begin{aligned}
 &= (1 - \eta\alpha s_t)(1 + \tilde{c}_t) \left[ 1 - \alpha + \alpha(\gamma - \eta)s_t + \alpha \left( \eta - \frac{1}{\sigma} \right) q_t \right] = 1 + \tilde{c}_t - \eta\alpha s_t + \alpha(\gamma - \eta)s_t + \alpha \left( \eta - \frac{1}{\sigma} \right) q_t = \\
 &= 1 + \tilde{c}_t + \alpha\gamma s_t + \alpha \left( \eta - \frac{1}{\sigma} \right) q_t
 \end{aligned}$$

which yields

$$\tilde{y}_t = \tilde{c}_t + \alpha\gamma s_t + \alpha \left( \eta - \frac{1}{\sigma} \right) q_t = \{q_t = (1 - \alpha)s_t\} = \tilde{c}_t + \alpha\gamma s_t + \alpha \left( \eta - \frac{1}{\sigma} \right) (1 - \alpha)s_t = \tilde{c}_t + \frac{\alpha\omega}{\sigma} s_t \quad [29]$$

where  $\omega \equiv \sigma\gamma + (1 - \alpha)(\sigma\eta - 1)$ .

Condition [29] applies to all countries, therefore for country  $i$  it takes the form

$\tilde{y}_t^i = \tilde{c}_t^i + \frac{\alpha\omega}{\sigma} s_t^i$ . Summing up all countries, we obtain the world market clearing condition

$$\tilde{y}_t^* \equiv \int_0^1 \tilde{y}_t^i di = \int_0^1 \tilde{c}_t^i di \equiv \tilde{c}_t^* \quad [30]$$

where  $\tilde{y}_t^*$  and  $\tilde{c}_t^*$  are stationary world output and consumption (in log terms),

whereas the market clearing equality derives from the fact that  $\int_0^1 s_t^i di = 0$ .

Substituting the expression for  $c_t$  from [20] into [29] and using equality [30] implies

$$\tilde{y}_t = \tilde{c}_t + \frac{\alpha\omega}{\sigma} s_t = \left\{ (20) \Rightarrow \tilde{c}_t = \tilde{c}_t^* + \left( \frac{1 - \alpha}{\sigma} \right) s_t \right\} = \tilde{c}_t^* + \left( \frac{1 - \alpha}{\sigma} \right) s_t + \frac{\alpha\omega}{\sigma} s_t = \tilde{y}_t^* + \frac{1 - \alpha + \alpha\omega}{\sigma} s_t$$

or

$$\tilde{y}_t = \tilde{y}_t^* + \frac{1}{\sigma_\alpha} s_t \quad [31]$$

where  $\sigma_\alpha \equiv \frac{\sigma}{1 + \alpha(\omega - 1)} > 0$ .

Expressing [29] for  $t + 1$  and using Euler equation [10] to substitute for  $E_t\{c_{t+1}\}$  results in

$$\begin{aligned}
 E_t\{\tilde{y}_{t+1}\} &= E_t\{\tilde{c}_{t+1}\} + \frac{\alpha\omega}{\sigma} E_t\{s_{t+1}\} = \tilde{c}_t + \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\}) - \frac{1}{\sigma} E_t\{\Delta a_{t+1}\} + \frac{\alpha\omega}{\sigma} E_t\{s_{t+1}\} = \\
 &= \left\{ (30) \Rightarrow \tilde{c}_t = \tilde{y}_t - \frac{\alpha\omega}{\sigma} s_t \right\} = \tilde{y}_t - \frac{\alpha\omega}{\sigma} s_t + \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\}) - \frac{1}{\sigma} E_t\{\Delta a_{t+1}\} + \frac{\alpha\omega}{\sigma} E_t\{s_{t+1}\}
 \end{aligned}$$

or

$$\begin{aligned}
 \tilde{y}_t &= E_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma} (r_t - E_t \{ \pi_{t+1} \}) + \frac{1}{\sigma} E_t \{ \Delta a_{t+1} \} - \frac{\alpha \omega}{\sigma} E_t \{ \Delta s_{t+1} \} = \\
 &= \left\{ (15) \Rightarrow E_t \{ \pi_{t+1} \} = E_t \{ \pi_{H,t+1} + \alpha \Delta s_{t+1} \} \right\} = \\
 &= E_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma} (r_t - E_t \{ \pi_{H,t+1} + \alpha \Delta s_{t+1} \}) + \frac{1}{\sigma} E_t \{ \Delta a_{t+1} \} - \frac{\alpha \omega}{\sigma} E_t \{ \Delta s_{t+1} \} = \\
 &= E_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma} (r_t - E_t \{ \pi_{H,t+1} \}) + \frac{1}{\sigma} E_t \{ \Delta a_{t+1} \} - \frac{\alpha \Theta}{\sigma} E_t \{ \Delta s_{t+1} \} = \\
 &= \left\{ (32) \Rightarrow s_t = \sigma_a (\tilde{y}_t - \tilde{y}_t^*) \Rightarrow E_t \{ \Delta s_{t+1} \} = \sigma_a E_t \{ \Delta \tilde{y}_{t+1} - \Delta \tilde{y}_{t+1}^* \} \right\} \\
 &= E_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma_a (1 + \alpha \Theta)} (r_t - E_t \{ \pi_{H,t+1} \}) + \frac{1}{\sigma} E_t \{ \Delta a_{t+1} \} - \frac{\alpha \Theta \sigma_a (E_t \{ \Delta \tilde{y}_{t+1} - \Delta \tilde{y}_{t+1}^* \})}{\sigma_a (1 + \alpha \Theta)} = \\
 &= E_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma_a (1 + \alpha \Theta)} (r_t - E_t \{ \pi_{H,t+1} \}) + \frac{1}{\sigma} E_t \{ \Delta a_{t+1} \} - \frac{\alpha \Theta E_t \{ \Delta \tilde{y}_{t+1} \}}{(1 + \alpha \Theta)} + \frac{\alpha \Theta E_t \{ \Delta \tilde{y}_{t+1}^* \}}{(1 + \alpha \Theta)}.
 \end{aligned}$$

Rearranging yields

$$\begin{aligned}
 0 &= E_t \{ \Delta \tilde{y}_{t+1} \} - \frac{\alpha \Theta E_t \{ \Delta \tilde{y}_{t+1} \}}{(1 + \alpha \Theta)} - \frac{1}{\sigma_a (1 + \alpha \Theta)} (r_t - E_t \{ \pi_{H,t+1} \}) + \frac{1}{\sigma} E_t \{ \Delta a_{t+1} \} + \frac{\alpha \Theta E_t \{ \Delta \tilde{y}_{t+1}^* \}}{(1 + \alpha \Theta)}; \\
 0 &= E_t \{ \Delta \tilde{y}_{t+1} \} \left[ 1 - \frac{\alpha \Theta}{(1 + \alpha \Theta)} \right] - \frac{1}{\sigma_a (1 + \alpha \Theta)} (r_t - E_t \{ \pi_{H,t+1} \}) + \frac{1}{\sigma} E_t \{ \Delta a_{t+1} \} + \frac{\alpha \Theta E_t \{ \Delta \tilde{y}_{t+1}^* \}}{(1 + \alpha \Theta)} = \\
 &= E_t \{ \Delta \tilde{y}_{t+1} \} \frac{1}{(1 + \alpha \Theta)} - \frac{1}{\sigma_a (1 + \alpha \Theta)} (r_t - E_t \{ \pi_{H,t+1} \}) + \frac{1}{\sigma} E_t \{ \Delta a_{t+1} \} + \frac{\alpha \Theta E_t \{ \Delta \tilde{y}_{t+1}^* \}}{(1 + \alpha \Theta)}; \\
 0 &= E_t \{ \Delta \tilde{y}_{t+1} \} - \frac{1}{\sigma_a} (r_t - E_t \{ \pi_{H,t+1} \}) + \frac{1}{\sigma} (1 + \alpha \Theta) E_t \{ \Delta a_{t+1} \} + \alpha \Theta E_t \{ \Delta \tilde{y}_{t+1}^* \}; \\
 \tilde{y}_t &= E_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma_a} (r_t - E_t \{ \pi_{H,t+1} \}) + \frac{1}{\sigma} (1 + \alpha \Theta) E_t \{ \Delta a_{t+1} \} + \alpha \Theta E_t \{ \Delta \tilde{y}_{t+1}^* \} \quad [32]
 \end{aligned}$$

where  $\Theta \equiv (\sigma \gamma - 1) + (1 - \alpha)(\sigma \eta - 1) = \omega - 1$ .

### The supply side

Let  $Y_t \equiv \left[ \int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$  represent an index for aggregate domestic output. One

can derive a production function linking the aggregate domestic demand with aggregate employment. Market clearing in the labour market requires

$$N_t = \int_0^1 N_t(j) dj.$$

Expressing labour demand from the firm's production function as  $N_t(j) = Y_t(j) / A_t$  and plugging into the labour market clearing condition yields

$$N_t = \int_0^1 \frac{Y_t(j)}{A_t} dj = \int_0^1 \frac{Y_t(j)Y_t}{Y_t A_t} dj = \frac{Y_t}{A_t} \int_0^1 \frac{Y_t(j)}{Y_t} dj$$

or using [4]

$$N_t \equiv \frac{Y_t}{A_t} \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj.$$

Taking logs,

$$y_t = a_t + n_t - d_t$$

where  $d_t \equiv \log \int_0^1 (P_t(j)/P_t)^{-\varepsilon} dj$ . In a neighbourhood of the zero inflation steady state,  $d_t$  is equal to zero up to a first-order approximation, therefore the last relationship reduces to

$$y_t = a_t + n_t \quad [33].$$

To get the inflation equation, it is useful to rewrite [26] taking into account that the condition  $mc_{t+k|t} = mc_{t+k}$  holds for the Cobb-Douglas production function  $Y_t = A_t N_t$ . The condition implies equal marginal costs both for firms which reoptimise their price and for the rest.

$$\begin{aligned} \bar{p}_{H,t} - p_{H,t-1} &= (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \hat{mc}_{t+k} + (p_{H,t+k} - p_{H,t-1}) \} = \\ &= (1 - \beta\theta) \left[ \hat{mc}_t + (p_{H,t} - p_{H,t-1}) \right] + (1 - \beta\theta) \sum_{k=1}^{\infty} (\beta\theta)^k E_t \{ \hat{mc}_{t+k} + (p_{H,t+k} - p_{H,t-1}) \} \\ &= (1 - \beta\theta) \left[ \hat{mc}_t + \pi_{H,t} \right] + (1 - \beta\theta) \beta\theta \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \hat{mc}_{t+k+1} + (p_{H,t+k+1} - p_{H,t} + p_{H,t} - p_{H,t-1}) \} = \\ &= (1 - \beta\theta) \hat{mc}_t + (1 - \beta\theta) \pi_{H,t} + (1 - \beta\theta) \beta\theta \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \hat{mc}_{t+k+1} + (p_{H,t+k+1} - p_{H,t}) + (p_{H,t} - p_{H,t-1}) \} = \\ &= (1 - \beta\theta) \hat{mc}_t + \pi_{H,t} - \beta\theta \pi_{H,t} + \\ &+ (1 - \beta\theta) \beta\theta \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \hat{mc}_{t+k+1} + (p_{H,t+k+1} - p_{H,t}) \} + \underbrace{(p_{H,t} - p_{H,t-1})}_{\pi_{H,t}} (1 - \beta\theta) \beta\theta \sum_{k=0}^{\infty} (\beta\theta)^k = \\ &= (1 - \beta\theta) \hat{mc}_t + \pi_{H,t} - \beta\theta \pi_{H,t} + (1 - \beta\theta) \beta\theta \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \hat{mc}_{t+k+1} + (p_{H,t+k+1} - p_{H,t}) \} + \beta\theta \pi_{H,t} \end{aligned}$$

or

$$\bar{p}_{H,t} - p_{H,t-1} = (1 - \beta\theta)\beta\theta \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \hat{m}c_{t+k+1} + (p_{H,t+k+1} - p_{H,t}) + (1 - \beta\theta)\hat{m}c_t + \pi_{H,t} \} \quad [33'].$$

Rewriting [26] for  $(\bar{p}_{H,t+1} - p_{H,t})$  yields

$$\bar{p}_{H,t+1} - p_{H,t} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_{t+1} \{ \hat{m}c_{t+k+1} + (p_{H,t+k+1} - p_{H,t}) \}.$$

Estimating expectation gives

$$E_t(\bar{p}_{H,t+1} - p_{H,t}) = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \hat{m}c_{t+k+1} + (p_{H,t+k+1} - p_{H,t}) \} \quad [33''].$$

Plugging [33''] into [33'] yields

$$\bar{p}_{H,t} - p_{H,t-1} = \beta\theta E_t(\bar{p}_{H,t+1} - p_{H,t}) + (1 - \beta\theta)\hat{m}c_t + \pi_{H,t} \quad [34].$$

Using [22], equation [34] implies

$$\frac{1}{1 - \theta} \pi_{H,t} = \beta\theta E_t \left( \frac{1}{1 - \theta} \pi_{H,t+1} \right) + (1 - \beta\theta)\hat{m}c_t + \pi_{H,t}$$

or

$$\frac{1}{1 - \theta} \pi_{H,t} - \pi_{H,t} = \frac{\theta}{1 - \theta} \pi_{H,t} = \beta\theta E_t \left( \frac{1}{1 - \theta} \pi_{H,t+1} \right) + (1 - \beta\theta)\hat{m}c_t.$$

This yields domestic inflation as a function of deviations of marginal cost from its steady state value

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \lambda \hat{m}c_t \quad [35]$$

where

$$\lambda \equiv \frac{(1 - \theta)(1 - \beta\theta)}{\theta}.$$

Equation [35] implies that inflation of domestically produced goods is not affected by parameters referring to the open economy. Conversely, real marginal cost as a function of domestic output in the open economy does differ from the closed economy case which results from the wedge between output and consumption, and between domestic and consumer prices. To show this is the case, let us first define real marginal cost (in terms of deviation from the steady-state) as a difference between real wage and marginal product of labour,  $mpn_t$ ,

$$\hat{m}c_t = (w_t - p_{H,t}) - mpn_t.$$

Rearranging and plugging definition of marginal product of labour consistent with [33], we obtain

$$\hat{mc}_t = (w_t - p_t) + \underbrace{(p_t - p_{H,t})}_{=(14)=\alpha_t} - a_t = \{(11) \Rightarrow w_t - p_t = \sigma \tilde{c}_t + \varphi n_t + a_t\} = \sigma \tilde{c}_t + \varphi n_t + a_t + \alpha s_t - a_t$$

or

$$\begin{aligned} \hat{mc}_t &= \sigma \tilde{c}_t + \varphi n_t + \alpha s_t = \sigma \left( \tilde{c}_t^* + \left( \frac{1-\alpha}{\sigma} \right) s_t \right) + \varphi (y_t - a_t) + \alpha s_t = \sigma \tilde{c}_t^* + (1-\alpha) s_t + \varphi (y_t - a_t) + \alpha s_t = \\ &= \sigma \tilde{y}_t^* + \varphi \tilde{y}_t + s_t \end{aligned} \quad [36]$$

where the first equality makes use of [20] and [33], but [30] is applied in the last one.

Using [31] to substitute for  $s_t$  in the previous expression yields the real marginal cost as a function of domestic output and world output

$$\hat{mc}_t = \sigma \tilde{y}_t^* + \varphi \tilde{y}_t + \sigma_\alpha (\tilde{y}_t - \tilde{y}_t^*) = (\sigma_\alpha + \varphi) \tilde{y}_t + (\sigma - \sigma_\alpha) \tilde{y}_t^* \quad [37].$$

Finally, under flexible prices, where  $\hat{mc}_t = 0$  for all  $t$ , the natural level of output in the open economy can be derived from [37]

$$0 = (\sigma_\alpha + \varphi) \tilde{y}_t^n + (\sigma - \sigma_\alpha) \tilde{y}_t^* \quad [38]$$

or rearranging

$$\tilde{y}_t^n = - \frac{(\sigma - \sigma_\alpha) \tilde{y}_t^*}{(\sigma_\alpha + \varphi)};$$

$$(\sigma - \sigma_\alpha) = \sigma_\alpha (1 + \alpha(\omega - 1)) - \sigma_\alpha = \sigma_\alpha (1 + \alpha(\omega - 1) - 1) = \sigma_\alpha \alpha(\omega - 1) = \sigma_\alpha \alpha \Theta$$

yields

$$\tilde{y}_t^n = \Gamma_* \tilde{y}_t^* \quad [39]$$

$$\text{where } \Gamma_* \equiv - \frac{\alpha \Theta \sigma_\alpha}{\sigma_\alpha + \varphi}.$$

### A simplified version

In the paper, we estimate a simplified version of Gali and Monacelli (2005) model presented above where  $\varphi = 0$ ,  $\eta = 1$ ,  $\gamma = 1$ , and  $1/\sigma = \tau$ . The open economy model consists of a forward-looking IS equation and the New Keynesian Phillips curve. Monetary policy is defined by an interest rate rule, whereas the exchange rate is introduced via the CPI equation given that PPP holds.

First, the Phillips curve is derived. Subtracting [38] from [37] yields

$$\hat{m}c_t = (\sigma_\alpha + \varphi)\tilde{y}_t + (\sigma - \sigma_\alpha)\tilde{y}_t^* - (\sigma_\alpha + \varphi)\tilde{y}_t^n - (\sigma - \sigma_\alpha)\tilde{y}_t^{*n} = (\sigma_\alpha + \varphi)(\tilde{y}_t - \tilde{y}_t^n) \quad [40].$$

Substituting [40] and  $\varphi = 0$  into [35] gives

$$\pi_{H,t} = \beta E_t \{\pi_{H,t+1}\} + \lambda \hat{m}c_t = \beta E_t \{\pi_{H,t+1}\} + \lambda \sigma_\alpha (\tilde{y}_t - \tilde{y}_t^n) \quad [41].$$

Recalling that  $\sigma_\alpha \equiv \frac{\sigma}{1 + \alpha(\omega - 1)}$  and  $\omega - 1 = \Theta = (\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1)$  which, under assumption  $\eta = 1$ ,  $\gamma = 1$ , becomes

$$\omega - 1 = \left(\frac{1}{\tau} - 1\right) + (1 - \alpha)\left(\frac{1}{\tau} - 1\right) = \left(\frac{1 - \tau}{\tau}\right)(2 - \alpha) \quad [42]$$

and plugging the latter into the definition for  $\sigma_\alpha$  yields

$$\sigma_\alpha = \frac{1/\tau}{1 + \alpha\left(\frac{1 - \tau}{\tau}\right)(2 - \alpha)} = \frac{1}{\tau + \alpha(1 - \tau)(2 - \alpha)} \quad [43].$$

Substituting [43] into [41] leads to the New Keynesian Phillips curve in terms of domestic goods inflation

$$\pi_{H,t} = \beta E_t \{\pi_{H,t+1}\} + \lambda \sigma_\alpha (\tilde{y}_t - \tilde{y}_t^n) = \beta E_t \{\pi_{H,t+1}\} + \frac{\lambda}{\tau + \alpha(1 - \tau)(2 - \alpha)} (\tilde{y}_t - \tilde{y}_t^n) \quad [44].$$

To derive [44] via the CPI inflation we use [15]. In contrast to Gali and Monacelli (2005) we define terms of trade as the relative price of exports in terms of imports, therefore CPI and domestic inflation are now related in the form

$$\pi_{H,t} = \pi_t + \alpha \Delta s_t$$

and for  $t + 1$

$$E_t \{\pi_{H,t+1}\} = E_t \{\pi_{t+1}\} + \alpha E_t \{\Delta s_{t+1}\}.$$

Substituting the last two expressions for  $\pi_{H,t}$  and  $E_t \{\pi_{H,t+1}\}$  into [44] results in

$$\pi_t + \alpha \Delta s_t = \beta (E_t \{\pi_{t+1}\} + \alpha E_t \{\Delta s_{t+1}\}) + \frac{\lambda}{\tau + \alpha(1 - \tau)(2 - \alpha)} (\tilde{y}_t - \tilde{y}_t^n)$$

or rearranging

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \alpha \beta E_t \{\Delta s_{t+1}\} - \alpha \Delta s_t + \frac{\lambda}{\tau + \alpha(2 - \alpha)(1 - \tau)} (\tilde{y}_t - \tilde{y}_t^n),$$

where the natural rate of output follows from [39]

$$\tilde{y}_t^n = \Gamma_* \tilde{y}_t^* = -\frac{\alpha \Theta \sigma_\alpha}{\sigma_\alpha + \varphi} \tilde{y}_t^* = -\alpha \Theta \tilde{y}_t^* = -\frac{\alpha(2 - \alpha)(1 - \tau)}{\tau} \tilde{y}_t^*.$$

Next, the IS curve is derived. Plugging  $\Theta = \omega - 1 = \left(\frac{1 - \tau}{\tau}\right)(2 - \alpha)$ ,

$\frac{1}{\sigma_\alpha} = \tau + \alpha(1 - \tau)(2 - \alpha)$ , and  $E_t \{\pi_{H,t+1}\} = E_t \{\pi_{t+1}\} + \alpha E_t \{\Delta s_{t+1}\}$  into [32] yields

$$\begin{aligned} \tilde{y}_t &= E_t \{\tilde{y}_{t+1}\} - (\tau + \alpha(1 - \tau)(2 - \alpha))(r_t - E_t \{\pi_{t+1} + \alpha \Delta s_{t+1}\}) + \tau \left(1 + \alpha \left(\frac{1 - \tau}{\tau}\right)(2 - \alpha)\right) E_t \{\Delta a_{t+1}\} + \alpha(2 - \alpha) \left(\frac{1 - \tau}{\tau}\right) E_t \{\Delta \tilde{y}_{t+1}^*\} \\ &= E_t \{\tilde{y}_{t+1}\} - (\tau + \alpha(1 - \tau)(2 - \alpha))(r_t - E_t \{\pi_{t+1} + \alpha \Delta s_{t+1}\}) + (\tau + \alpha(1 - \tau)(2 - \alpha)) E_t \{\Delta a_{t+1}\} + \alpha(2 - \alpha) \left(\frac{1 - \tau}{\tau}\right) E_t \{\Delta \tilde{y}_{t+1}^*\} \\ &= E_t \{\tilde{y}_{t+1}\} - (\tau + \alpha(1 - \tau)(2 - \alpha))(r_t - E_t \{\pi_{t+1} + \alpha \Delta s_{t+1}\} - E_t \{\Delta a_{t+1}\}) + \alpha(2 - \alpha) \left(\frac{1 - \tau}{\tau}\right) E_t \{\Delta \tilde{y}_{t+1}^*\}. \end{aligned}$$

Given that  $z_t$  is the growth rate of the world technology process  $A_t$ , i.e.,  $\Delta a_t = z_t$ , where  $a_t \equiv \log A_t$ , and is described by the AR(1) dynamics  $z_t = \rho_z z_{t-1} + \varepsilon_t^z$ , we obtain

$$E_t \{\Delta a_{t+1}\} = E_t \{z_{t+1}\} = \rho_z z_t.$$

Plugging the last derivation into the expression for  $\tilde{y}_t$  yields the IS equation

$$\tilde{y}_t = E_t \{\tilde{y}_{t+1}\} - (\tau + \alpha(1 - \tau)(2 - \alpha))(r_t - E_t \{\pi_{t+1} + \alpha \Delta s_{t+1}\} - \rho_z z_t) + \alpha(2 - \alpha) \left(\frac{1 - \tau}{\tau}\right) E_t \{\Delta \tilde{y}_{t+1}^*\}.$$

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