Short-Term Forecasting of GDP at the Bank of Latvia

Andrejs Bessonovs
Set of models:
- Traditional bridge equations;
- Bridge equations in state-space form;
- Dynamic factor models.
Operative indicators

- GDP data are available at quarterly frequency and become available with a lag:
  - Flash estimate ~ 2 months;
  - Official release ~ 3 months.

- Instead, most data relating GDP are available faster and at monthly frequency:
  - Money aggregate M3;
  - Industrial production;
  - Retail turnover, etc.
Dataset

- Real-time GDP database by monthly breakdown (monthly revisions) of expenditure and production side.
- Monthly indicators on economic activity including:
  - industrial production;
  - retail turnover;
  - exports, imports;
  - inflation;
  - money aggregates;
  - unemployment, vacancies;
  - taxes, etc.
- Business and consumer surveys.
Aggregated vs. disaggregated approach

- Three approaches:
  - GDP at aggregated level using monthly indicators;
  - GDP by expenditure side:
    \[ Y_{(expenditure)} = C + G + I + X + M; \]
  - GDP by output side:
    \[ Y_{(output)} = AB + CDE + F + G + I + HJKO + LMN + TS; \]
- Each component of expenditure and output basis has its own set of monthly indicators with appropriate economic meaning.
GDP forecasting using traditional bridge equations
Bridge equations describe the correlation between quarterly variables such as GDP (or its components) and monthly indicators.

Monthly indicators are converted to quarter frequency in line with their characteristics as stock or flow variables.

Then dependent variable is regressed on monthly indicators in quarterly frequency.
Concept of bridge equations

\[ y_{tq} = \rho(L)y_{tq} + \sum_{j=1}^{k} \delta_j(L)x_{j,tq}^{mq} + \varepsilon_{tq} \]

- \( y_{tq} \) – GDP quarterly growth;
- \( x^{mq} \) – set of monthly indicators converted to quarterly frequency;
- \( k \) – number of indicators.
GDP interpolation and short-term forecasting using bridge equations in state-space form
The use of bridge equations in state-space form helps to find correlations between quarterly GDP data and monthly indicators on a monthly basis.

Two equations:
- Transition equation: unobservable monthly GDP growth depends on operative monthly indicators;
- Measurement equation: sum of 3 months should be equal to the GDP quarter value.

Solved by Kalman filter.
State-space form

Lagged GDP growth

Monthly GDP growth

Operative monthly indicators

\[
\begin{pmatrix}
\Delta \ln y_{t+1}^m \\
\Delta \ln y_t^m \\
\Delta \ln y_{t-1}^m \\
\Delta \ln y_{t-2}^m \\
e_{t+1}
\end{pmatrix}
= 
\begin{pmatrix}
\xi & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\Delta \ln y_t^m \\
\Delta \ln y_{t-1}^m \\
\Delta \ln y_{t-2}^m \\
\Delta \ln y_{t-3}^m \\
e_t
\end{pmatrix}
+ 
\begin{pmatrix}
\beta_1 & \cdots & \beta_N \\
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
\Delta \ln x_{1,t}^m \\
\vdots \\
\Delta \ln x_{N,t}^m
\end{pmatrix}
+ 
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
\sigma^2
\end{pmatrix}
\]

\[
\Delta \ln y_t^Q = \frac{1}{3} \Delta \ln y_t^m + \frac{2}{3} \Delta \ln y_{t-1}^m + \Delta \ln y_{t-2}^m + \frac{2}{3} \Delta \ln y_{t-3}^m + \frac{1}{3} \Delta \ln y_{t-4}^m + \xi_t
\]

Quarterly GDP growth is linked to the monthly GDP growth rates
GDP growth forecast using monthly GDP estimates
State-space form advantages and disadvantages

- **Advantages:**
  - Helps to estimate monthly GDP.

- **Disadvantages:**
  - Using short time series Kalman filter results are unstable;
  - Results are sensitive to set of variables one use in state-space form.
GDP forecasting using dynamic factor models
Dynamic factor models

- Regression analysis usually uses 4-5 variables at most:
  - Technical difficulties (number of variables cannot exceed number of observations);
  - Models become unstable or inefficient.
- However, there are a lot of variables which contain important information about economic activity.
- Factor models allow to use all that information without losing too much degrees of freedom.
There exist few unobservable factors, which explain most of economic indicators’ fluctuations.

Those factors are independent from each other.

We reduce all necessary information about economic activity into unobservable factors.

We are able to calculate unobservable components using Principal Components Analysis.
Stock-Watson dynamic factor model

Quarterly GDP growth \hspace{0.5cm} \text{Set of factors} \hspace{0.5cm} \text{Lagged GDP growth}

\[ y_{t+1} = \beta(L)F_t + \gamma(L)y_t + u_{t+1} \]

\[ X_t = \Lambda F_t + \xi_t \]

Set of indicators \hspace{1cm} \text{Idiosyncratic component}
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Database $X$ could be divided into two subsets:
- $X^{NA}$ – missing observations;
- $X^{OBS}$ – available observations.

We can estimate missing observations using expectation-maximization (EM) algorithm.
Expectation-maximization algorithm

- Stop, when changes in $F$ are small:

$$X = \begin{pmatrix} X^{OBS} \\ X^{NA} \end{pmatrix} \rightarrow F \rightarrow X = \begin{pmatrix} X^{OBS} \\ \hat{X}^{NA} \end{pmatrix}$$

$$X^{OBS} = \Lambda F + \epsilon$$

$$\hat{X}^{NA} = \hat{\Lambda} F$$

Factor analysis

$\hat{\mu}$

$F$
Unobservable factors

1.factor

2.factor

3.factor

4.factor
While modelling and forecasting with factor models, one should consider the following:
- Number of unobservable factors;
- Number of lags of latent factors;
- Number of lags of endogenous variable.

We chose parameters, which maximize the forecasting ability of the model (RMSFE).
Forecasting using dynamic factor models

- **1-step ahead:**
  \[ \hat{y}_{t+1}^1 = \alpha_1 + \beta_1(L)F_t + \gamma_1(L)y_t \]

- **h-steps ahead:**
  \[ \hat{y}_{t+h}^h = \alpha_h + \beta_h(L)F_t + \gamma_h(L)y_t \]
Dynamic factor models forecasts

- Next 4 quarters forecasts (model: 1 factor, 1 factor lag, none GDP lags)
Factor models advantages and disadvantages

- **Advantages:**
  - Factor models allow to use large datasets;
  - Using the same dataset one could forecast necessary macroeconomic variable, not even GDP.

- **Disadvantages:**
  - There is little economic interpretation for latent factors and equations;
  - Factor model tracks only past observations therefore predictability of the model is limited when structural breaks occur;
  - It is difficult to determine number of variables in dataset. Even more, the greater number of variables does not necessary improve model’s predictability.
Comparison of models’ forecasting ability
Comparing models’ forecasting ability

- There are 9 models for short-term forecast – which one to use?
- Start to look at out-of-sample forecast
  - 2/3 of sample - actual values, 1/3 - out of sample forecast.
- RMSFE indicates the forecasting performance of the model in the past:

\[
RMSFE = \sqrt{\frac{1}{T} \sum_{i=1}^{T} \left( y_i - \hat{y}_i^F \right)^2}
\]

Forecasting error
Forecasting ability: aggregated approach

GDP root mean squared forecasting error (RMSFE) (pp.)
2004Q4-2009Q1

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<th>Factor</th>
<th>Combination</th>
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* Forecast combination is just a simple average of individual models
Forecasting ability: disaggregated by expenditure

1 quarter ahead GDP RMSFE (pp.) on expenditure basis, 2004Q4-2009Q1

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<th>Government consumption</th>
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Forecasting ability: disaggregated by output

1 quarter ahead GDP RMSFE (pp.) on output basis, 2004Q4-2009Q1

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Results of the short-term forecasting models are reported to the Board of the Bank of Latvia and Monetary Policy Department colleagues on a weekly basis.