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Short-Term Forecasting of GDP at the Bank of Latvia

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Short-term Forecasting of GDP at the Bank of Latvia

- Set of models:
  - – Traditional bridge equations;
  - – Bridge equations in state-space form;
  - – Dynamic factor models.

Operative indicators

- GDP data are available at quarterly frequency and become available with a lag:
  - Flash estimate ~ 2 months;
  - Official release ~ 3 months.
- Instead, most data relating GDP are available faster and at monthly frequency:
  - Money aggregate M3;
  - Industrial production;
  - Retail turnover, etc.

Dataset

- Real-time GDP database by monthly breakdown (monthly revisions) of expenditure and production side.
- Monthly indicators on economic activity including:
  - industrial production;
  - retail turnover;
  - exports, imports;
  - inflation;
  - money aggregates;
  - unemployment, vacancies;
  - taxes, etc.
- Business and consumer surveys.

Aggregated vs. disaggregated approach

- Three approaches:
  - GDP at aggregated level using monthly indicators;
  - GDP by expenditure side:

    $Y_{(expenditure)} = C + G + I + X + M;$
  - GDP by output side:

    $Y_{(output)} = AB + CDE + F + G + I + HJKO + LMN + TS;$
- Each component of expenditure and output basis has its own set of monthly indicators with appropriate economic meaning.

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GDP forecasting using traditional bridge equations

Concept of bridge equations

- Bridge equations describe the correlation between quarterly variables such as GDP (or its components) and monthly indicators.
- Monthly indicators are converted to quarter frequency in line with their characteristics as stock or flow variables.
- Then dependent variable is regressed on monthly indicators in quarterly frequency.

Concept of bridge equations

Quarterly GDP growth

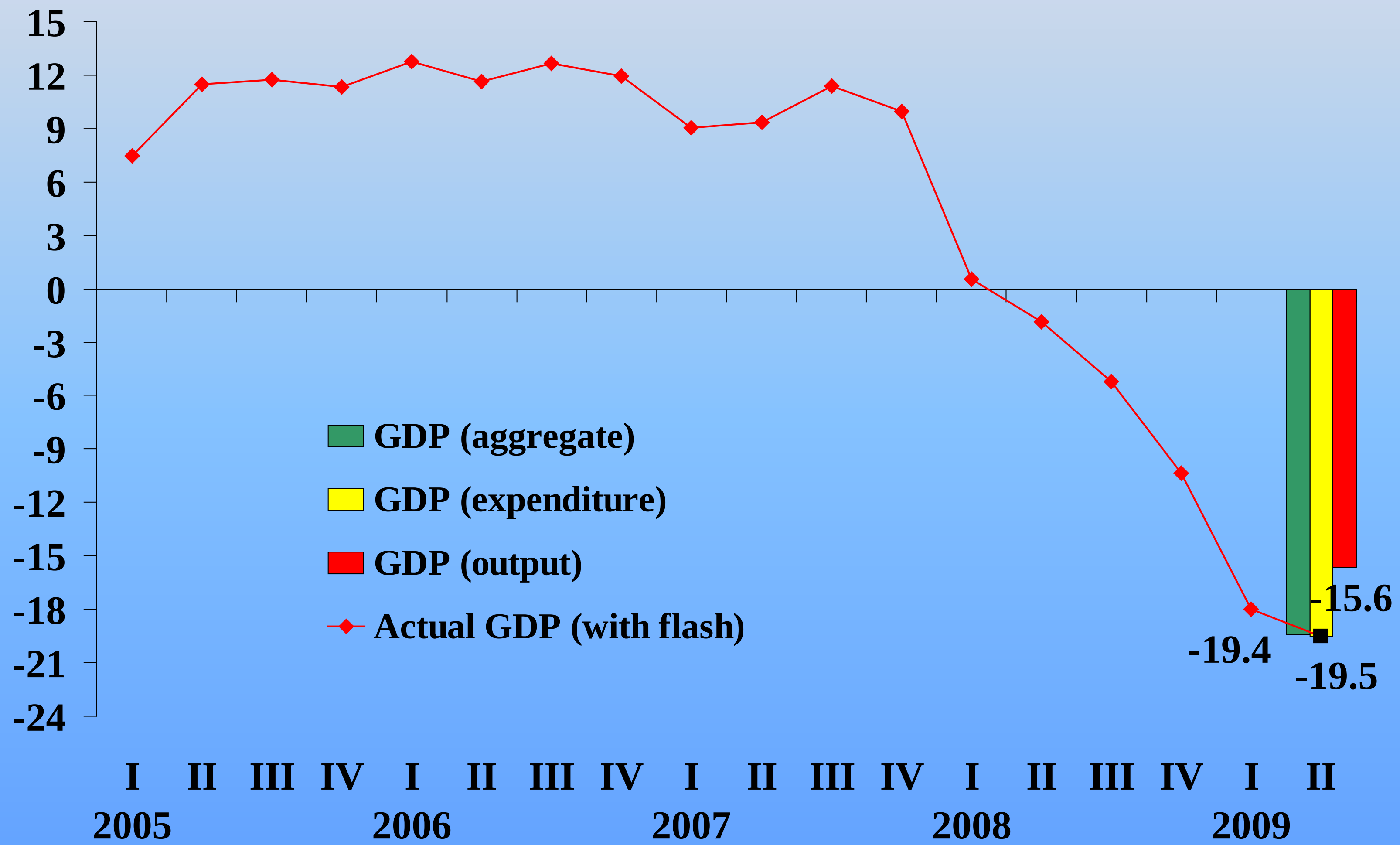
Lagged value of GDP growth

Monthly indicators

$$y_{t_q} = \rho(L) y_{t_q} + \sum_{j=1}^{k} \delta_j(L) x_{j,t_q}^{mq} + \varepsilon_{t_q}$$

- y_{tq} –GDP quarterly growth;
- x^{mq} – set of monthly indicators converted to quarterly frequency;
- k – number of indicators.

Bridge equations' forecasts



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GDP interpolation and short-term forecasting using bridge equations in state-space form

State-space form

- The use of bridge equations in state-space form helps to find correlations between quarterly GDP data and monthly indicators on a monthly basis.
- Two equations:
  - Transition equation: unobservable monthly GDP growth depends on operative monthly indicators;
  - Measurement equation: sum of 3 months should be equal to the GDP quarter value.
- Solved by Kalman filter.

State-space form

Lagged GDP growth

Monthly GDP growth

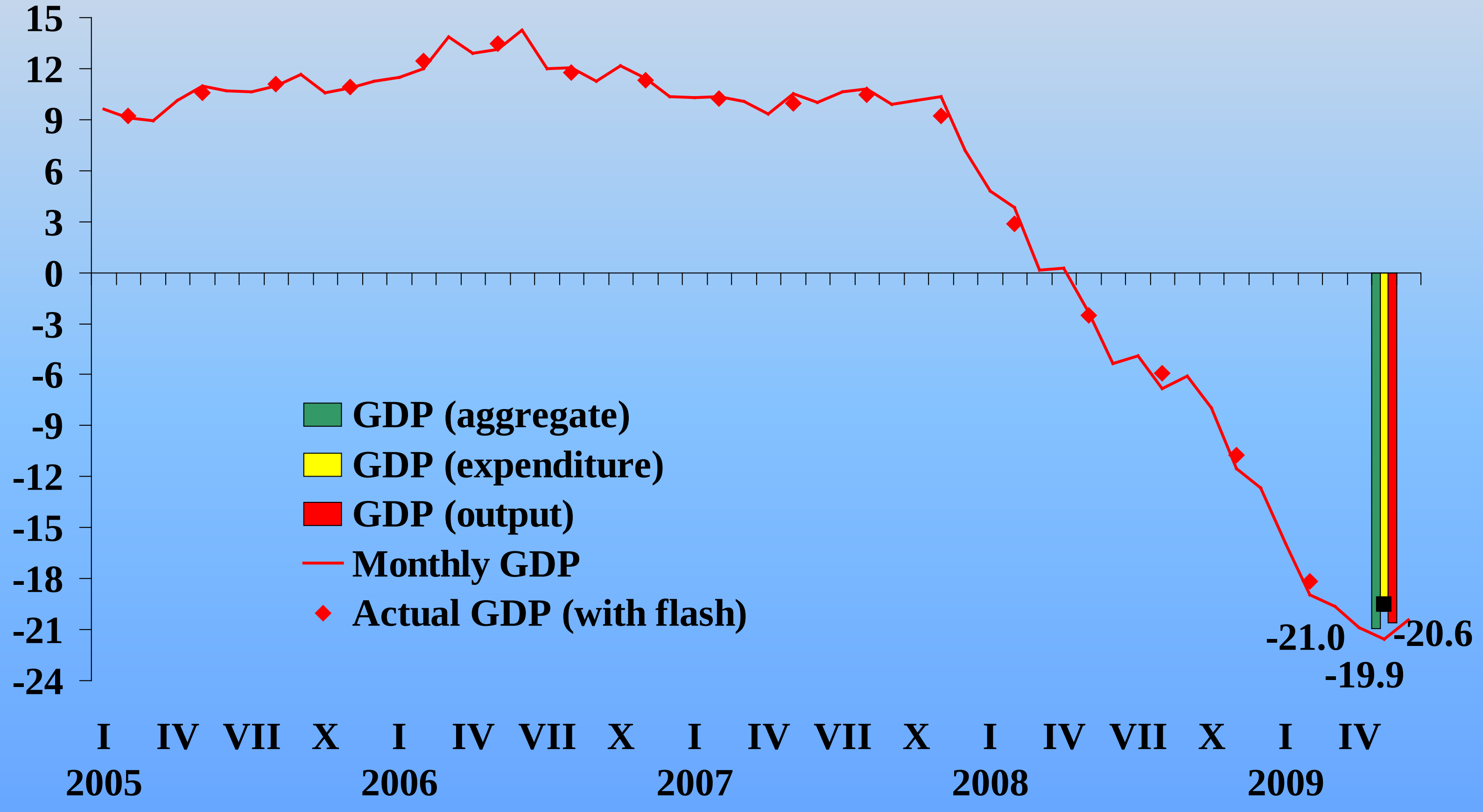
Operative monthly indicators

$$\begin{pmatrix} \Delta \ln y_{t+1}^m \\ \Delta \ln y_t^m \\ \Delta \ln y_{t-1}^m \\ \Delta \ln y_{t-2}^m \\ \Delta \ln y_{t-3}^m \\ e_{t+1} \end{pmatrix} = \begin{pmatrix} \zeta & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta \ln y_t^m \\ \Delta \ln y_{t-1}^m \\ \Delta \ln y_{t-2}^m \\ \Delta \ln y_{t-3}^m \\ \Delta \ln y_{t-4}^m \\ e_t \end{pmatrix} + \begin{pmatrix} \beta_1 & \cdots & \beta_N \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \Delta \ln x_{1,t}^m \\ \vdots \\ \Delta \ln x_{N,t}^m \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma^2 \end{pmatrix} u_{t+1}$$

$$\Delta \ln y_\tau^Q = \frac{1}{3}\Delta \ln y_t^m + \frac{2}{3}\Delta \ln y_{t-1}^m + \Delta \ln y_{t-2}^m + \frac{2}{3}\Delta \ln y_{t-3}^m + \frac{1}{3}\Delta \ln y_{t-4}^m + \xi_\tau$$

Quarterly GDP growth is linked to the monthly GDP growth rates

GDP growth forecast using monthly GDP estimates



State-space form advantages and disadvantages

- Advantages:
  - Helps to estimate monthly GDP.
- Disadvantages:
  - Using short time series Kalman filter results are unstable;
  - Results are sensitive to set of variables one use in state-space form.

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GDP forecasting using dynamic factor models

Dynamic factor models

- Regression analysis usually uses 4-5 variables at most:
  - Technical difficulties (number of variables cannot exceed number of observations);
  - Models become unstable or inefficient.
- However, there are a lot of variables which contain important information about economic activity.
- Factor models allow to use all that information without losing too much degrees of freedom.

Concept of Factor Models

- There exist few unobservable factors, which explain most of economic indicators' fluctuations.
- Those factors are independent from each other.
- We reduce all necessary information about economic activity into unobservable factors.
- We are able to calculate unobservable components using Principal Components Analysis.

Stock-Watson dynamic factor model

Quarterly GDP growth

Set of factors

Lagged GDP growth

$$y_{t+1} = \beta(L)F_t + \gamma(L)y_t + u_{t+1}$$

$$X_t = \Lambda F_t + \xi_t$$

Set of indicators

Idiosyncratic component

Incomplete datasets

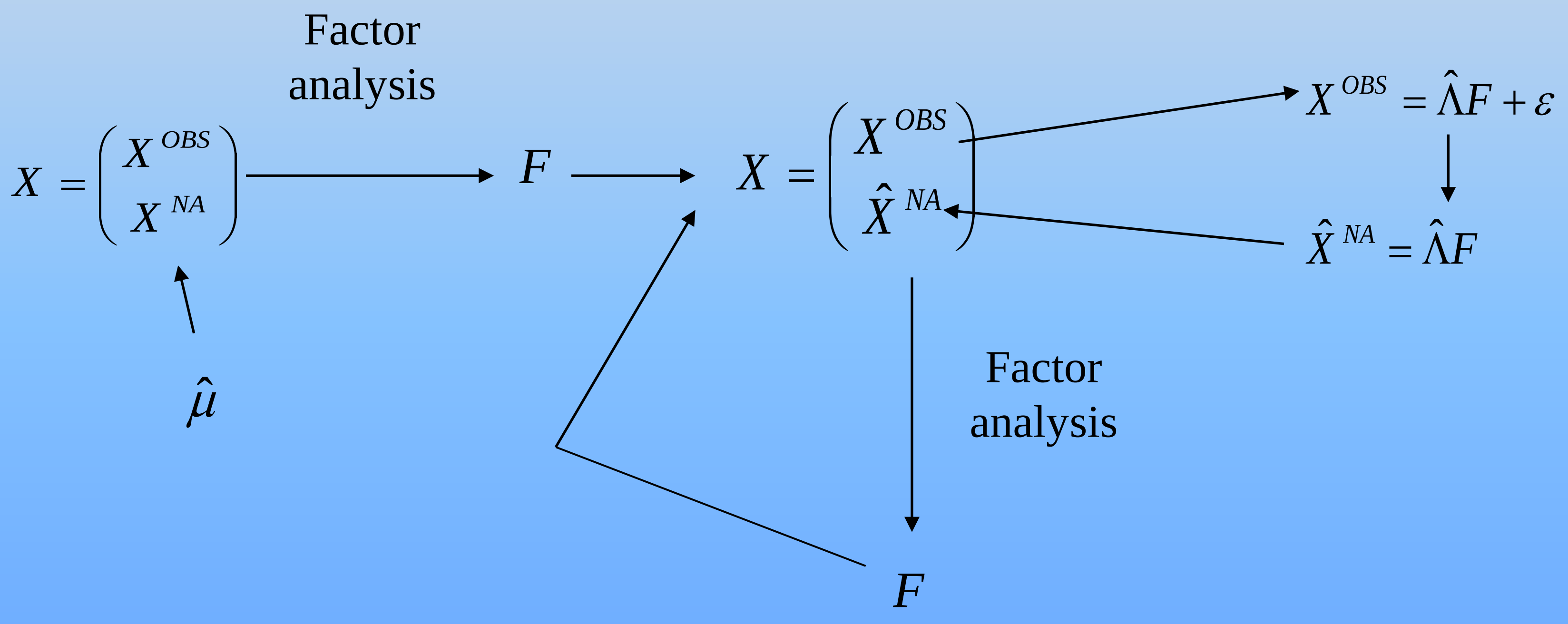
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
1999M07	-21.2	-9.9	N/A	N/A	-43.5	91.2	-30.1	N/A	N/A	N/A	111.0	172.0
1999M08	-21.1	-9.7	N/A	N/A	-44.0	91.3	-23.9	N/A	N/A	N/A	110.5	175.3
1999M09	-20.9	-9.6	N/A	N/A	-45.5	91.5	-22.5	N/A	N/A	N/A	112.0	180.0
1999M10	-11.8	-10.2	N/A	N/A	-44.5	96.0	-22.3	N/A	N/A	N/A	111.1	181.4
1999M11	-11.4	-10.2	N/A	N/A	-45.5	96.3	-18.3	N/A	N/A	N/A	113.3	183.3
1999M12	-11.1	-10.1	N/A	N/A	-46.0	96.5	-15.6	N/A	N/A	N/A	115.2	184.6
2000M01	-14.3	-2.7	N/A	N/A	-32.0	91.1	-16.7	1758	-13595	11068	114.4	186.4
2000M02	-14.0	-2.3	N/A	N/A	-32.5	90.9	-15.8	-47391	31978	14901	115.4	185.8
2000M03	-13.7	-2.4	N/A	N/A	-32.5	90.7	-17.3	-10534	11097	-24232	117.0	185.2
:	:	:	:	:	:	:	:	:	:	:	:	:
2001M04	-3.4	14.9	N/A	N/A	1.0	99.5	-2.4	-36353	16562	-8532	120.9	178.4
2001M05	-6.6	19.4	N/A	N/A	1.0	102.9	-3.0	473	-28502	8225	122.0	178.5
2001M06	-3.3	28.6	N/A	N/A	1.5	104.3	-10.1	-102918	137589	-13223	124.3	178.1
2001M07	1.7	13.8	N/A	N/A	N/A	103.7	-10.5	-47712	102368	-26735	123.6	179.2
2001M08	2.5	9.5	N/A	N/A	N/A	106.3	-6.2	51661	-58787	13167	121.1	181.2
2001M09	0.4	9.1	N/A	N/A	N/A	103.8	-9.6	5170	27891	4536	121.3	182.3
2001M10	-1.2	12.7	N/A	N/A	N/A	100.7	-7.0	-2212	46315	-22871	121.2	181.3
2001M11	-2.0	10.2	N/A	N/A	N/A	100.3	-7.6	143211	1625	-111120	122.0	180.9
2001M12	-5.5	4.3	N/A	N/A	N/A	98.8	-6.4	89525	109352	-28649	121.1	180.2
2002M01	3.9	7.4	N/A	N/A	N/A	104.2	-7.3	5434	10808	384	120.6	180.0
2002M02	3.9	-0.1	10.3	N/A	N/A	107.3	-7.0	-9898	-11061	13202	121.6	180.4
2002M03	6.2	4.5	6.9	N/A	N/A	110.8	-7.7	-22522	43591	-5050	121.1	180.7
2002M04	-1.9	9.1	8.9	N/A	N/A	109.5	-8.2	-53069	55865	11920	120.6	180.2
2002M05	-1.6	18.8	9.0	2.1	17.6	107.1	-8.9	-39135	64181	9461	119.3	179.7
2002M06	-1.5	21.2	9.1	9.8	17.3	107.5	-13.6	25102	-73701	10965	116.9	179.1
:	:	:	:	:	:	:	:	:	:	:	:	:
2009M05	-29.0	-44.2	-40.9	-41.5	-69.5	70.3	-31.2	14893	-276095	16535	105.8	171.1
2009M06	-27.7	-42.0	-40.5	-45.6	-70.7	70.2	-29.0	11527	-308734	99374	105.9	171.5
2009M07	-26.7	-34.0	-39.7	-38.4	-70.3	71.5	-28.7	N/A	N/A	N/A	N/A	N/A

Expectation-maximization algorithm

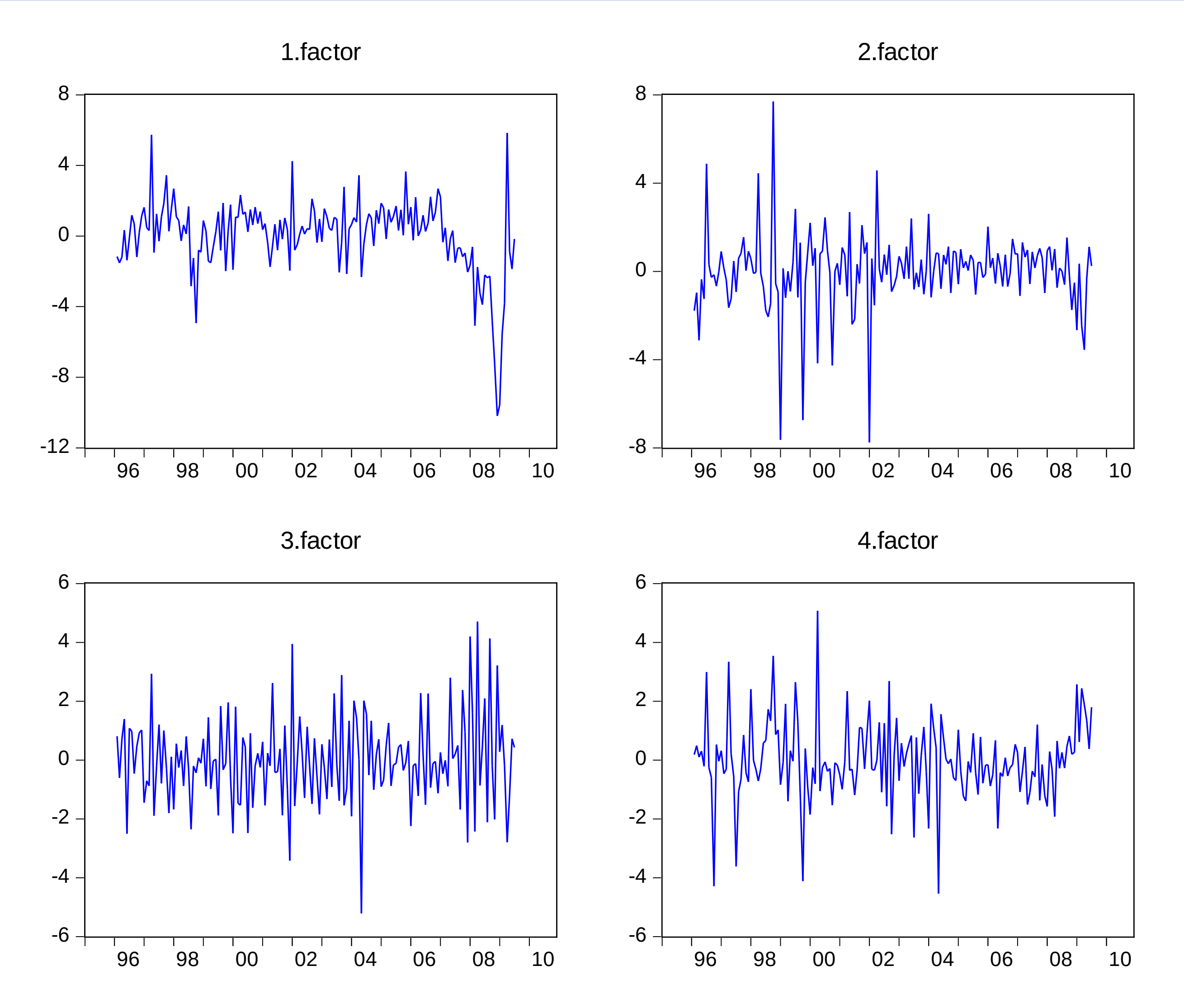
- Database X could be divided into two subsets:
  - – X^{NA} – missing observations;
  - – X^{OBS} – available observations.
- We can estimate missing observations using expectation-maximization (EM) algorithm.

Expectation-maximization algorithm

- Stop, when changes in *F* are small:



Unobservable factors



Forecasting using dynamic factor models

- While modelling and forecasting with factor models, one should consider the following:
  - Number of unobservable factors;
  - Number of lags of latent factors;
  - Number of lags of endogenous variable.
- We chose parameters, which maximize the forecasting ability of the model (RMSFE).

Forecasting using dynamic factor models

- 1-step ahead:

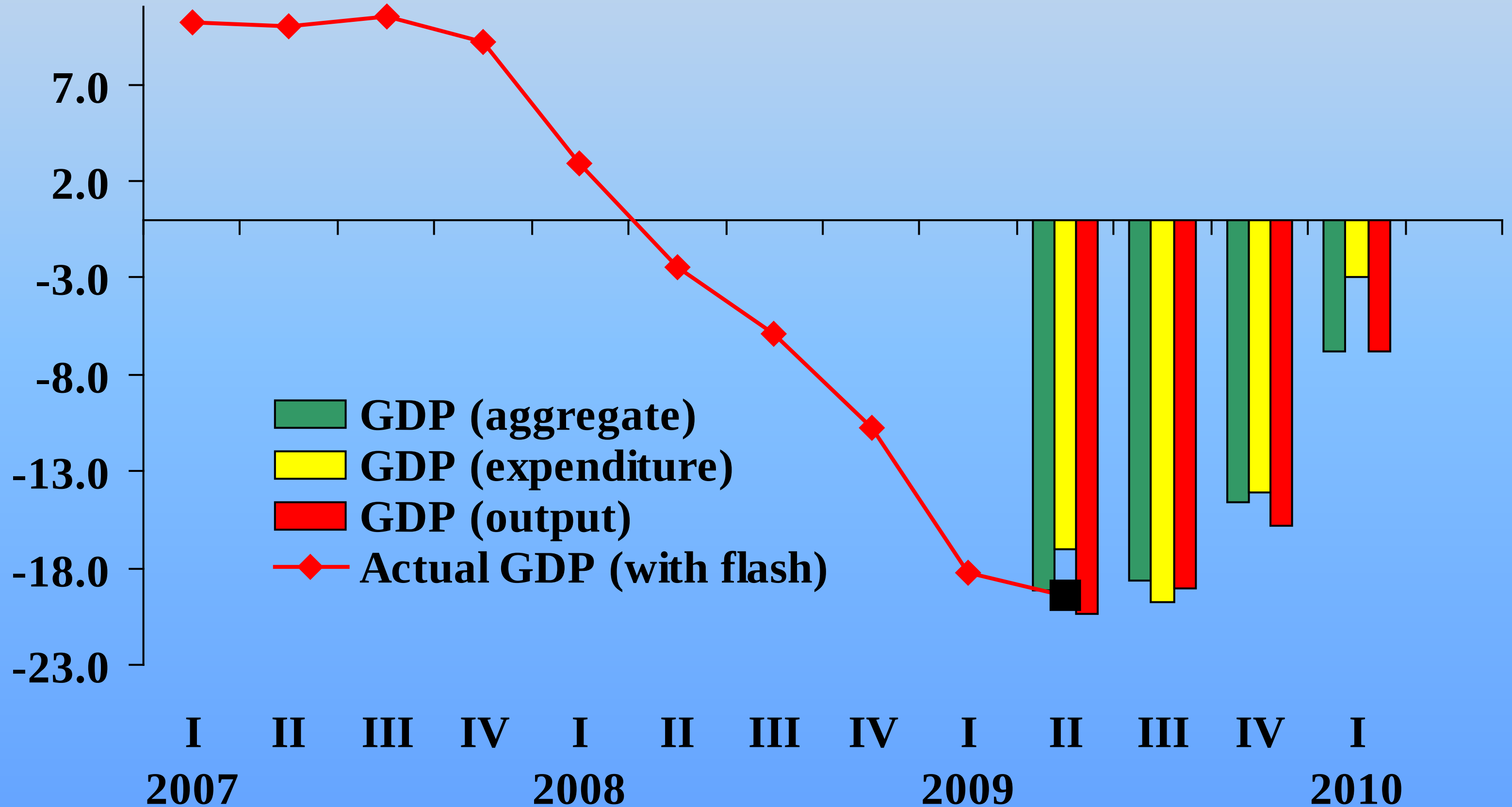
$$\hat{y}_{t+1}^{1} = \alpha_1 + \beta_1(L)F_t + \gamma_1(L)y_t$$

- h-steps ahead:

$$\hat{y}_{t+h}^{h} = \alpha_h + \beta_h(L)F_t + \gamma_h(L)y_t$$

Dynamic factor models forecasts

- Next 4 quarters forecasts (model: 1 factor, 1 factor lag, none GDP lags)



Factor models advantages and disadvantages

- Advantages:
  - Factor models allow to use large datasets;
  - Using the same dataset one could forecast necessary macroeconomic variable, not even GDP.
- Disadvantages:
  - There is little economic interpretation for latent factors and equations;
  - Factor model tracks only past observations therefore predictability of the model is limited when structural breaks occur;
  - It is difficult to determine number of variables in dataset. Even more, the greater number of variables does not necessary improve model's predictability.

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Comparison of models' forecasting ability

Comparing models' forecasting ability

- There are 9 models for short-term forecast – which one to use?
- Start to look at out-of-sample forecast
  - 2/3 of sample - actual values, 1/3 - out of sample forecast.
- RMSFE indicates the forecasting performance of the model in the past:

$$RMSFE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}\underbrace{\left(y_i - \hat{y}_i^F\right)}_{\text{Forecasting error}}^2}$$

Forecasting ability: aggregated approach

GDP root mean squared forecasting error (RMSFE) (pp.)
2004Q4-2009Q1

Horizon	Traditional bridge	Bridge in state-space	Factor	Combination
+Q1	2.61	2.58	2.56	2.52
+Q2			3.45	3.92
+Q3			6.34	6.72
+Q4			8.13	8.63

* Forecast combination is just a simple average of individual models

Forecasting ability: disaggregated by expenditure

1 quarter ahead GDP RMSFE (pp.) on expenditure basis, 2004Q4-2009Q1

Model	Y(expenditure)	Private consumption	Government consumption	Investment	Exports	Imports
Traditional Bridge	4.34	5.11	9.7	14.55	4.12	4.74
Bridge in state-space	4.1	5.59	10.93	15.13	3.88	4.74
Factor	2.12	5.32	9.6	12.86	3.69	7.02

Forecasting ability: disaggregated by output

1 quarter ahead GDP RMSFE (pp.) on output basis, 2004Q4-2009Q1

Model	Y(output)	AB	CDE	F	G	I	HJKO	LMN	TS
Traditional bridge	2.77	5.25	2.86	4.94	3.53	7.34	3.39	2.8	14.52
Bridge in state-space	2.52	4.46	2.99	5.9	3.51	8.26	3.28	5.01	15.56
Factor	2.58	5.64	2.26	5.65	3.29	9.22	3.15	3.96	16.53

Practical use

- Results of the short-term forecasting models are reported to the Board of the Bank of Latvia and Monetary Policy Department colleagues on a weekly basis.